第三章　空间力系

§3－1　空间汇交力系

当空间力系中各力作用线汇交于一点时，称其为空间汇交力系.

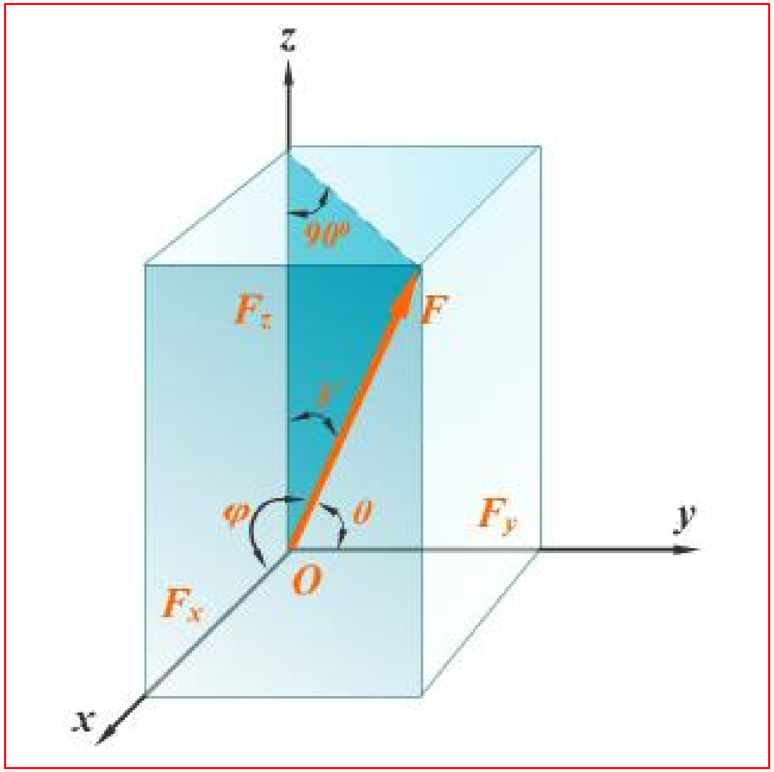
一. 力在直角坐标轴上的投影

直接投影法

$$F_x = F\cos\varphi$$

$$F_y = F\cos\theta$$

$$F_z = F\cos\gamma$$



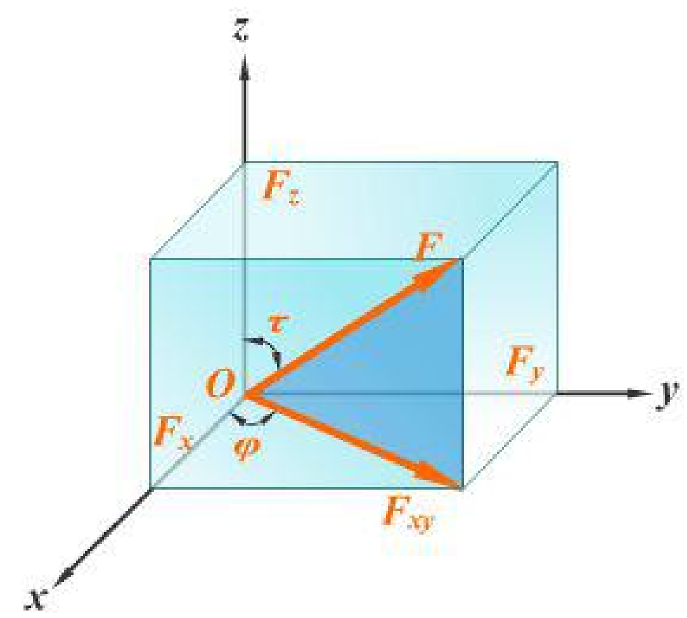
间接（二次）投影法

$$F_{xy} = F\sin\tau$$

$$F_x = F\sin\tau\cos\varphi$$

$$F_y = F\sin\tau\sin\varphi$$

$$F_z = F\cos\tau$$



二. 空间汇交力系的合力与平衡条件

空间汇交力系的合力 $\vec{F}_{\mathrm{R}} = \sum \vec{F}_i$

合矢量（力）投影定理

$$F_{\mathrm{R}x} = \sum F_{ix} = \sum F_x \quad F_{\mathrm{R}y} = \sum F_{iy} = \sum F_y \quad F_{\mathrm{R}z} = \sum F_{iz} = \sum F_z$$

合力的大小 $F_{\mathrm{R}} = \sqrt{(\sum F_x)^2 + (\sum F_y)^2 + (\sum F_z)^2}$

方向余弦

$$\cos(\vec{F}_{\mathrm{R}}, \vec{i}) = \frac{\sum F_x}{F_{\mathrm{R}}} \quad \cos(\vec{F}_{\mathrm{R}}, \vec{j}) = \frac{\sum F_y}{F_{\mathrm{R}}} \quad \cos(\vec{F}_{\mathrm{R}}, \vec{k}) = \frac{\sum F_z}{F_{\mathrm{R}}}$$

空间汇交力系的合力等于各分力的矢量和，合力的作用线通过汇交点.

空间汇交力系平衡的充分必要条件是：

该力系的合力等于零，即 $\vec{F}_{\mathrm{R}} = 0$

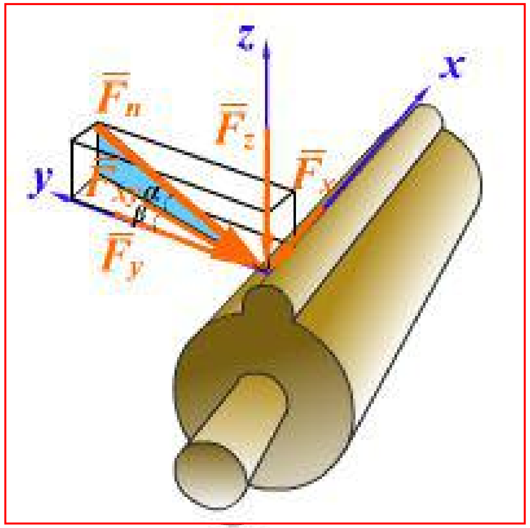
$$\boxed{\sum F_x = 0} \qquad \boxed{\sum F_y = 0} \qquad \boxed{\sum F_z = 0}$$

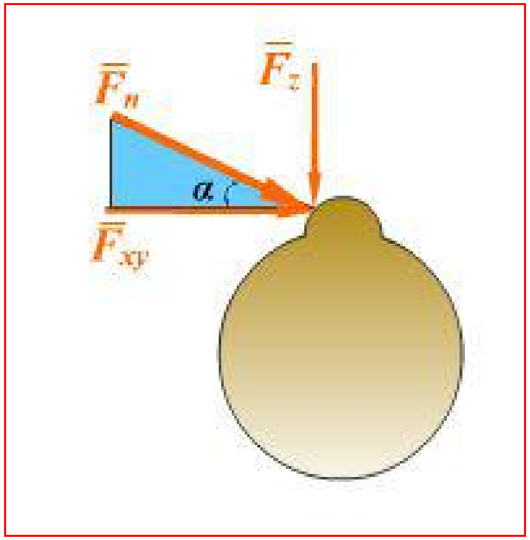
--称为空间汇交力系的平衡方程

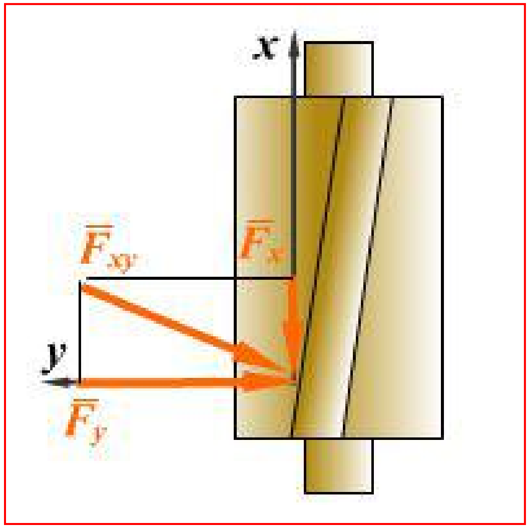
空间汇交力系平衡的充要条件：该力系中所有各力在三个坐标轴上的投影的代数和分别为零.

例3-1 已知：$\vec{F}_n\,,\beta\,,\alpha$

求：力 $\vec{F}_n$ 在三个坐标轴上的投影.







解：

$$F_z = -F_n \sin\alpha \qquad F_{xy} = F_n \cos\alpha$$

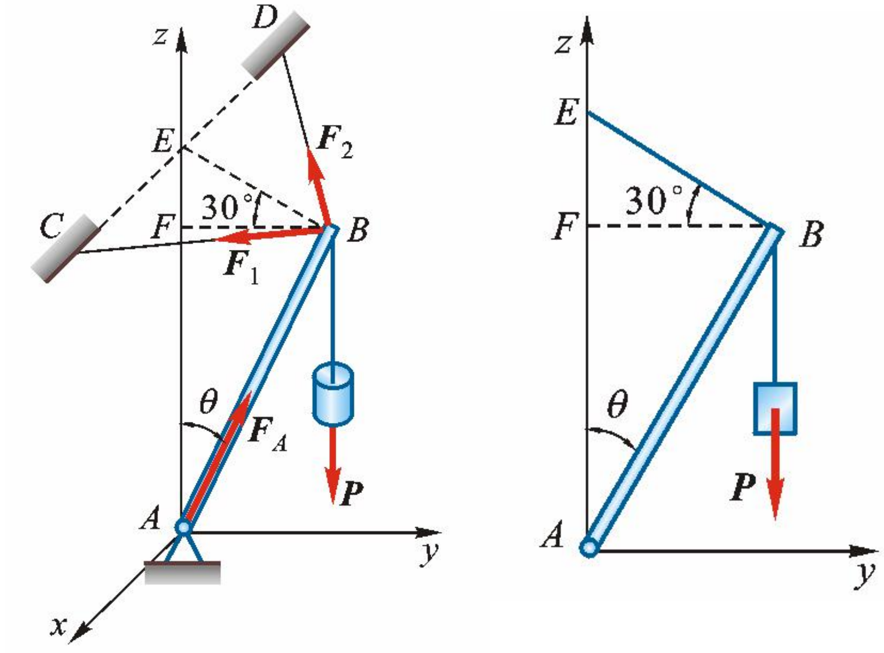
$$F_x = -F_{xy} \sin\beta = -F_n \cos\alpha \sin\beta$$

$$F_y = -F_{xy} \cos\beta = -F_n \cos\alpha \cos\beta$$

例3-2　已知：物重P=10kN，CE=EB=DE；$\theta = 30^0$

求：杆受力及绳拉力

解： 画受力图，列平衡方程



$$\sum F_x = 0$$

$$F_1 \sin 45^\circ - F_2 \sin 45^\circ = 0$$

$$\sum F_y = 0$$

$$F_A \sin 30^\circ - F_1 \cos 45^\circ \cos 30^\circ - F_2 \cos 45^\circ \cos 30^\circ = 0$$

$$\sum F_z = 0$$

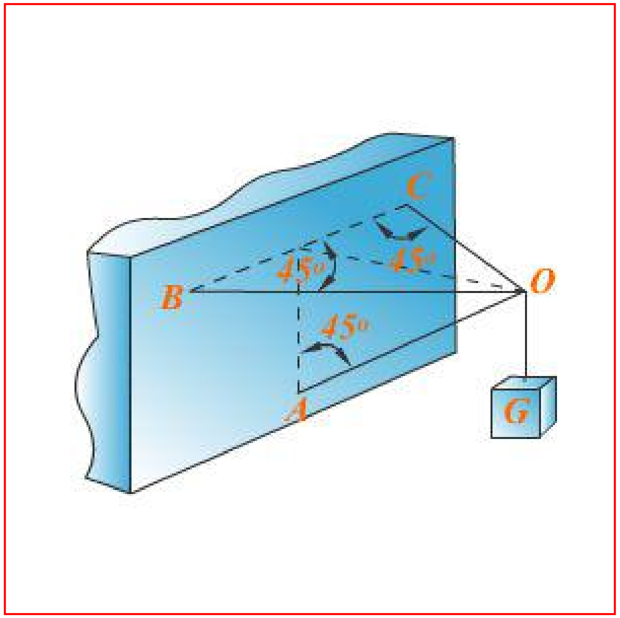
$$F_1 \cos 45^\circ \sin 30^\circ + F_2 \cos 45^\circ \sin 30^\circ + F_A \cos 30^\circ - P = 0$$

$\Rightarrow$ $F_1 = F_2 = 3.54\text{kN}$　$F_A = 8.66\text{kN}$

例3-3

已知：$P=1000\text{N}$，各杆重不计.

求：三根杆所受力.



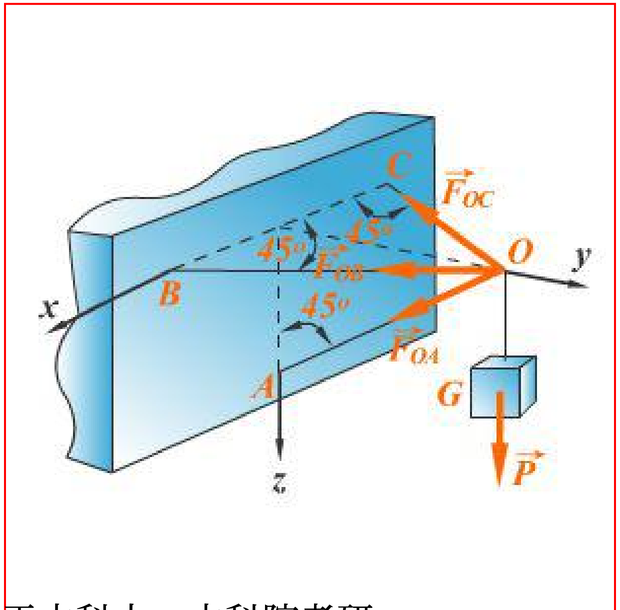
解： 各杆均为二力杆，取球铰O，画受力图。

$\sum F_x = 0 \qquad F_{OB}\sin 45^\circ - F_{OC}\sin 45^\circ = 0$

$\sum F_y = 0$

$-F_{OB}\cos 45^\circ - F_{OC}\cos 45^\circ - F_{OA}\cos 45^\circ = 0$

$\sum F_z = 0 \qquad F_{OA}\sin 45^\circ + P = 0$



$F_{OA} = -1414\text{N} \quad F_{OB} = F_{OC} = 707\text{N}$（拉）

§3－2　力对点的矩和力对轴的矩

一. 力对点的矩以矢量表示 ——力矩矢

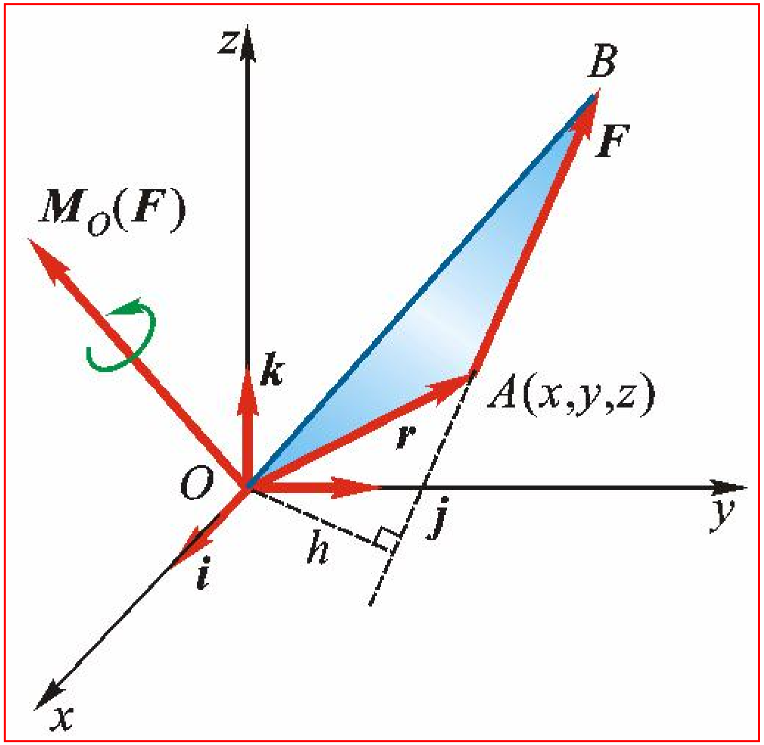
三要素:

（1）大小:力 $\vec{F}$ 与力臂的乘积

（2）方向:转动方向

（3）作用面：力矩作用面.

$$\vec{M}_O(\vec{F})=\vec{r}\times\vec{F}$$



$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \qquad \vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M}_O(\vec{F}) = (\vec{r} \times \vec{F}) = (x\vec{i} + y\vec{j} + z\vec{k}) \times (F_x\vec{i} + F_y\vec{j} + F_z\vec{k})$$

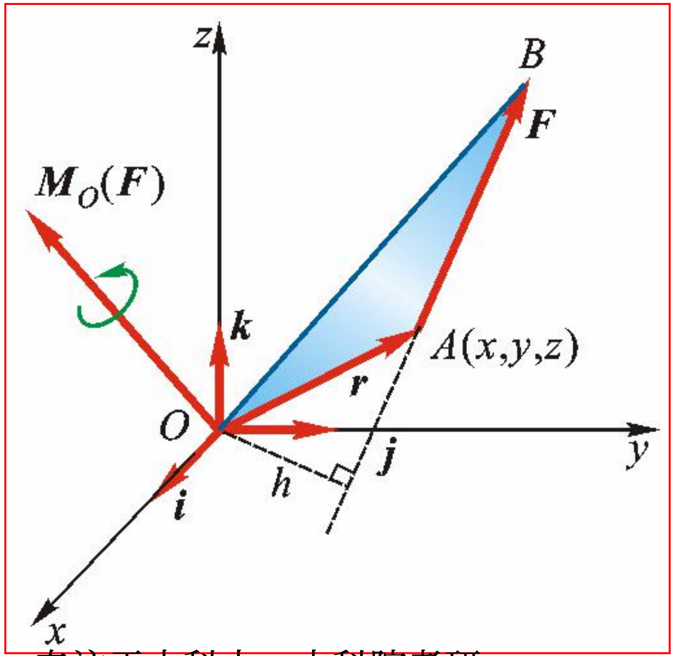
$$= (yF_z - zF_y)\vec{i} + (zF_x - xF_z)\vec{j} + (xF_y - yF_x)\vec{k}$$

力对点 O 的矩在三个坐标轴上的投影为

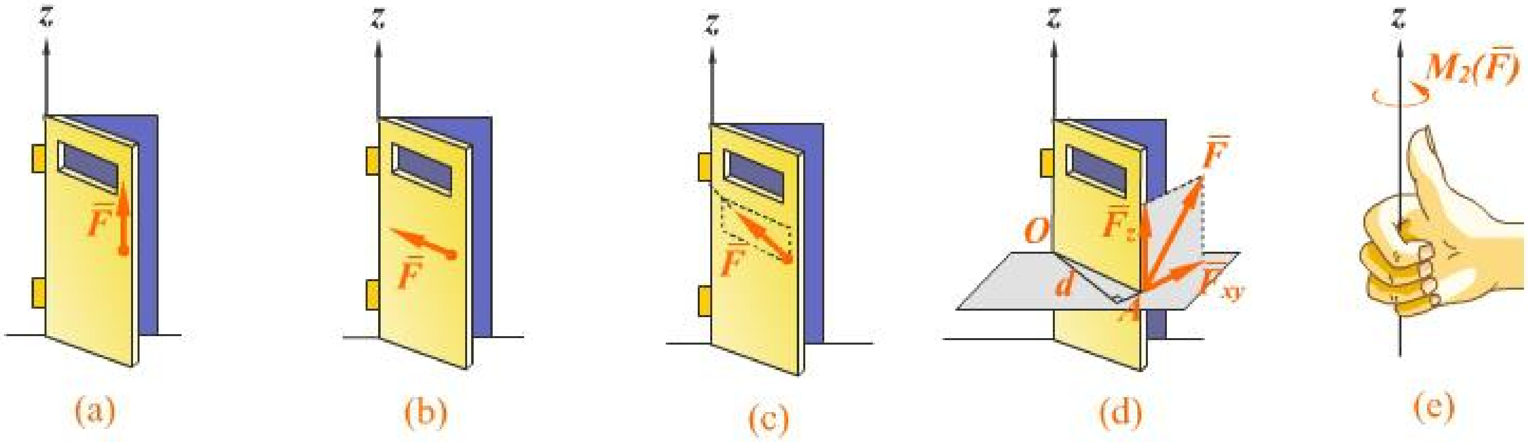
$$\left[\vec{M}_O(\vec{F})\right]_x = yF_z - zF_y$$

$$\left[\vec{M}_O(\vec{F})\right]_y = zF_x - xF_z$$

$$\left[\vec{M}_O(\vec{F})\right]_z = xF_y - yF_x$$



二. 力对轴的矩



$$\boxed{M_z(\vec{F})=M_O(\vec{F}_{xy})=\pm F_{xy}\cdot h}$$

力与轴相交或与轴平行（力与轴在同一平面内），力对该轴的矩为零.

三. 力对点的矩与力对过该点的轴的矩的关系

$$M_x(\vec{F}) = M_x(\vec{F}_x) + M_x(\vec{F}_y) + M_x(\vec{F}_z) = F_z \cdot y - F_y \cdot z$$

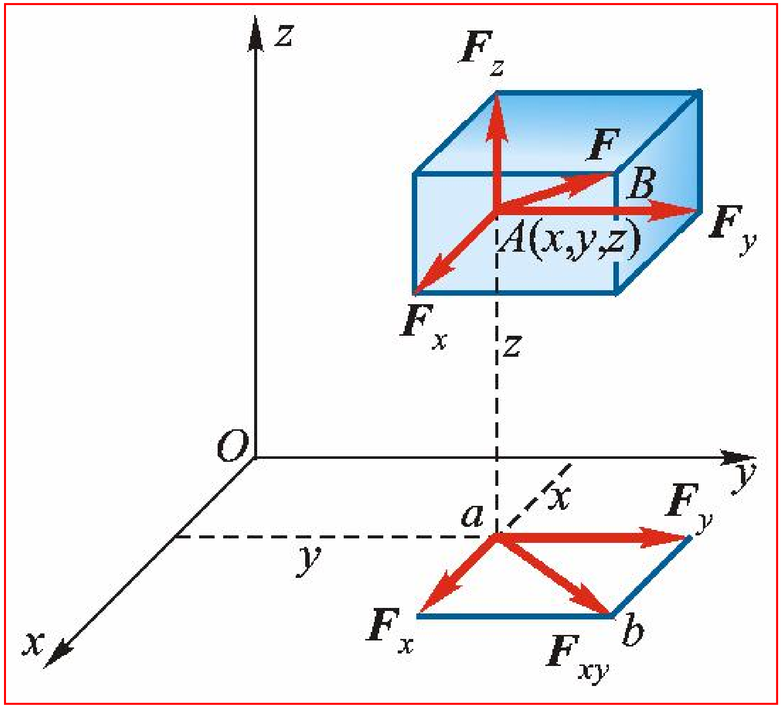
$$M_y(\vec{F}) = M_y(\vec{F}_x) + M_y(\vec{F}_y) + M_y(\vec{F}_z) = F_x \cdot z - F_z \cdot x$$

$$M_z(\vec{F}) = F_y \cdot x - F_x \cdot y$$

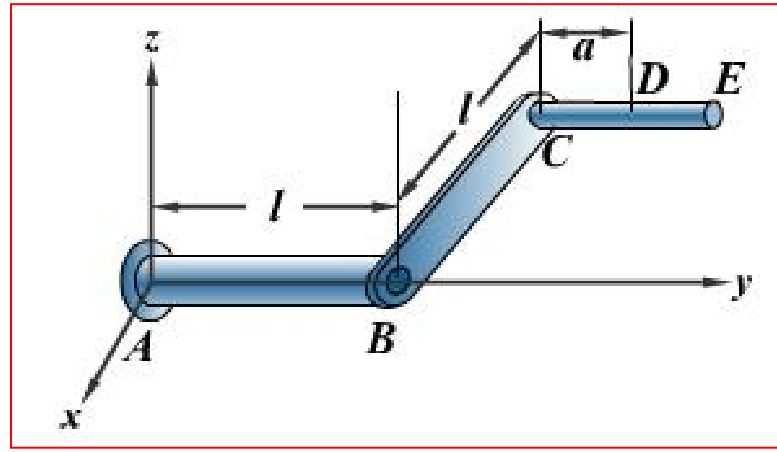
$$\left[\vec{M}_O(\vec{F})\right]_x = yF_z - zF_y = M_x(\vec{F})$$

$$\left[\vec{M}_O(\vec{F})\right]_y = zF_x - xF_z = M_y(\vec{F})$$

$$\left[\vec{M}_O(\vec{F})\right]_z = xF_y - yF_x = M_z(\vec{F})$$



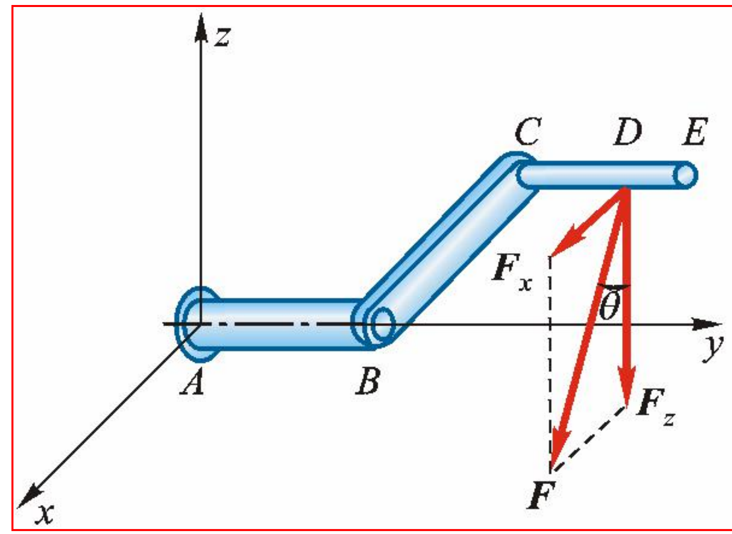
例3-4



已知： F, l, a, θ

求：$M_x(\vec{F}), M_y(\vec{F}), M_z(\vec{F})$

解： 把力 $\vec{F}$ 分解如图



$$M_x(\vec{F}) = -F(l+a)\cos\theta$$

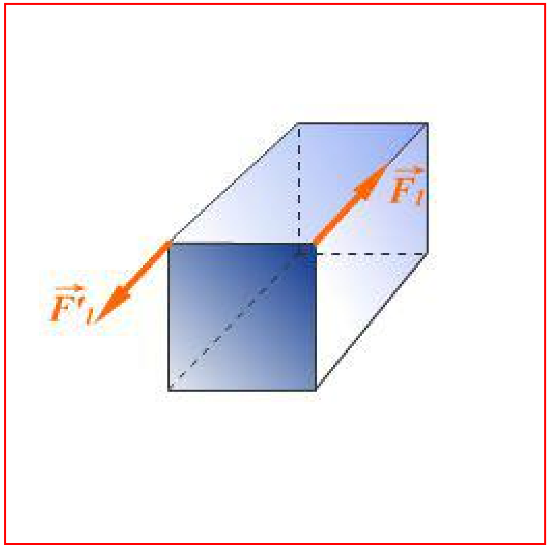
$$M_y(\vec{F}) = -Fl\cos\theta$$

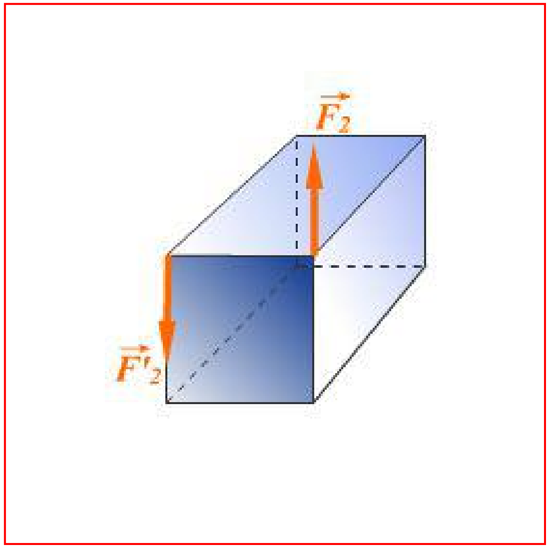
$$M_z(F) = -F(l+a)\sin\theta$$

§3－3　空间力偶

一. 力偶矩以矢量表示——力偶矩矢

$$F_1 = F_2 = F_1' = F_2'$$



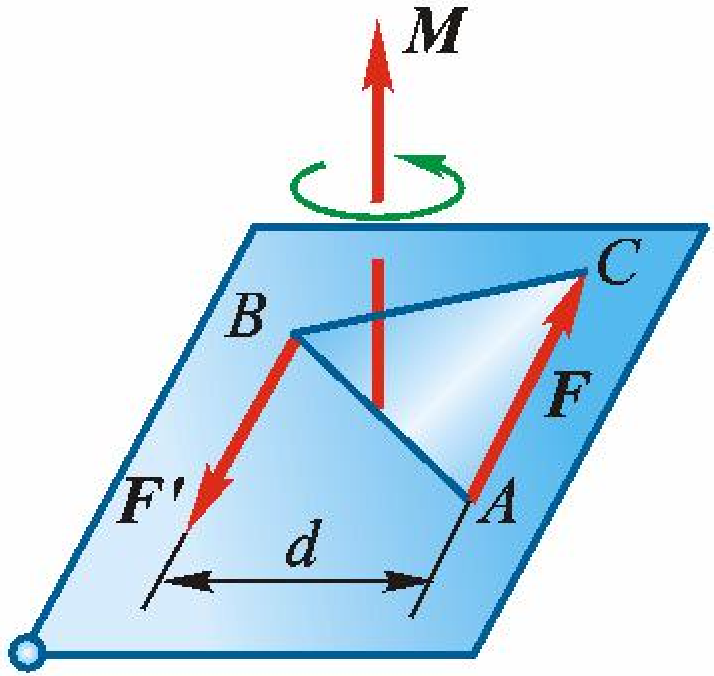


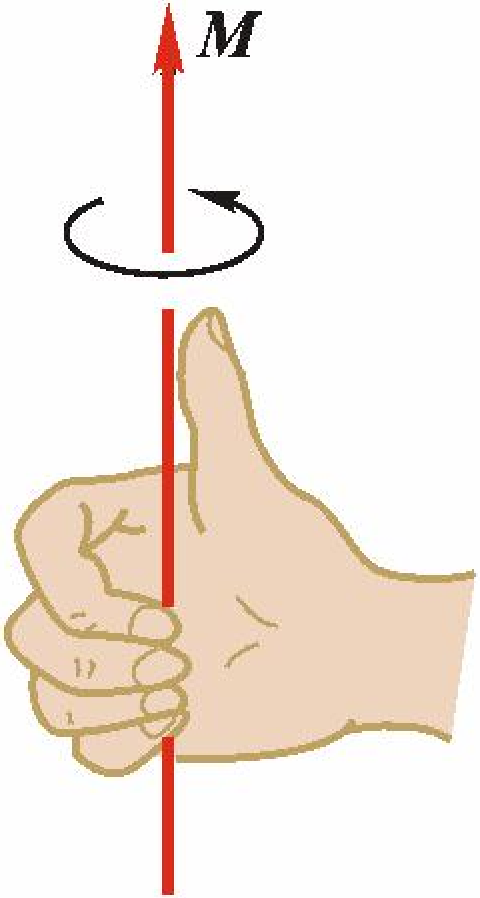
空间力偶的三要素

（1）　大小：力与力偶臂的乘积；

（2）　方向：转动方向；

（3）　作用面：力偶作用面。

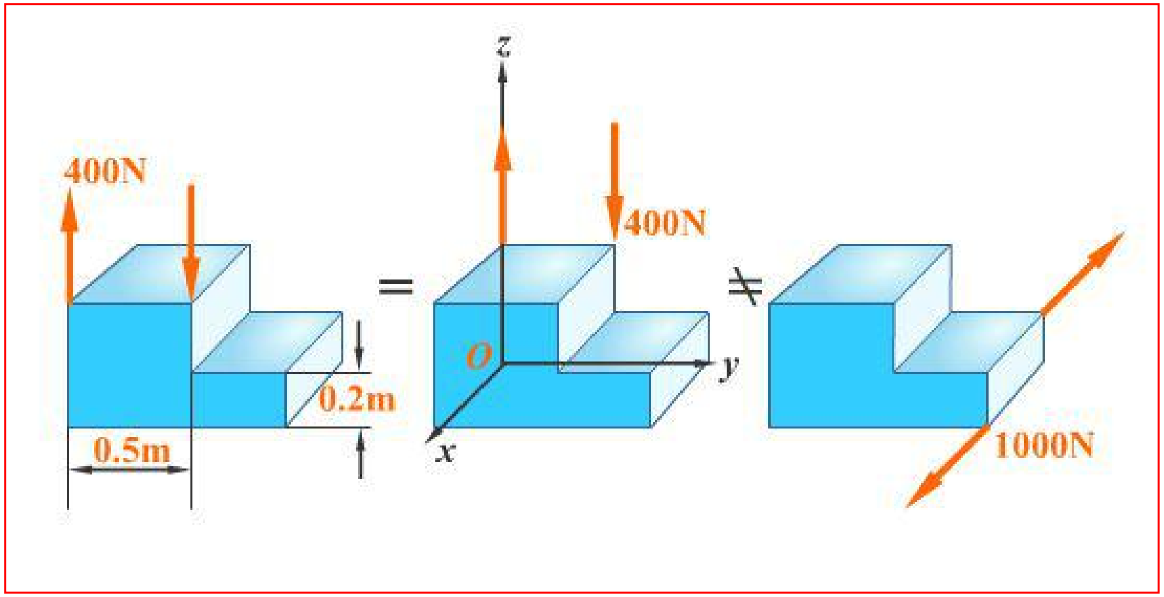




$$\vec{M} = \vec{r}_{BA} \times \vec{F}$$

二. 力偶的等效定理

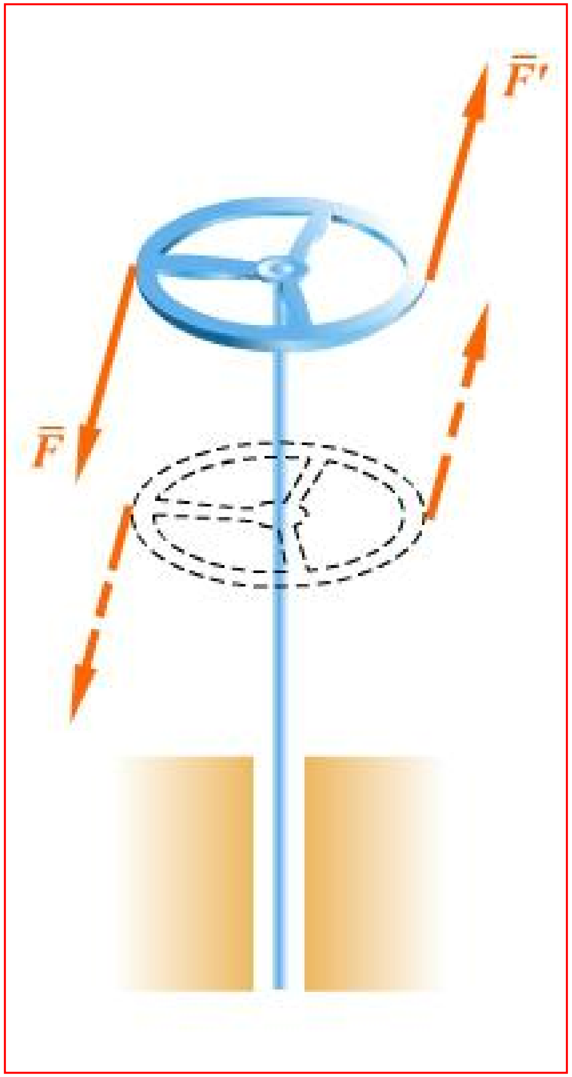
实例



空间力偶的等效定理：作用在同一刚体上的两个力偶，如果其力偶矩相等，则它们彼此等效。

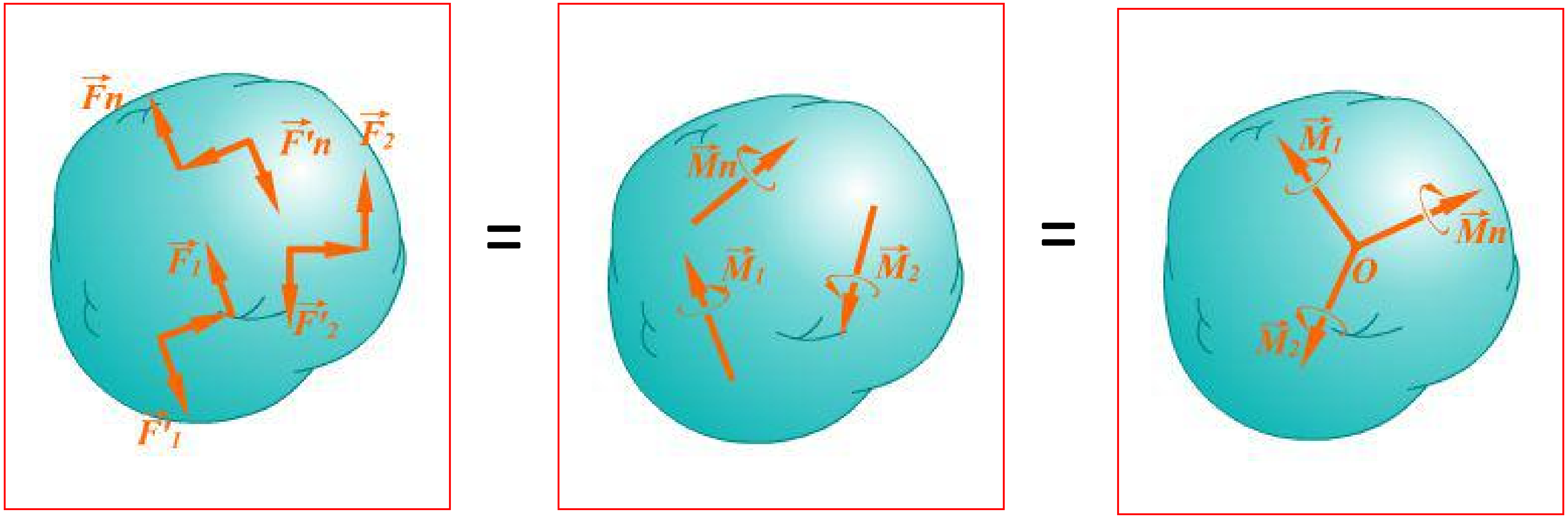
空间力偶可以平移到与其作用面平行的任意平面上而不改变力偶对刚体的作用效果.

只要保持力偶矩不变，力偶可在其作用面内任意移转，且可以同时改变力偶中力的大小与力偶臂的长短，对刚体的作用效果不变.



力偶矩矢是自由矢量

三．力偶系的合成与平衡条件



$$\vec{M}_1=\vec{r}_1\times\vec{F}_1,\vec{M}_2=\vec{r}_2\times\vec{F}_2,\ldots\ldots,\vec{M}_n=\vec{r}_n\times\vec{F}_n$$

$$\boxed{\vec{M}=\sum\vec{M}_i}$$

$\vec{M}$ 为合力偶矩矢，等于各分力偶矩矢的矢量和.

$$M_x = \sum M_x \,, M_y = \sum M_y \,, M_z = \sum M_z$$

合力偶矩矢的大小和方向余弦

$$M = \sqrt{(\sum M_x)^2 + (\sum M_y)^2 + (\sum M_z)^2}$$

$$\cos\theta = \frac{\sum M_x}{M} \qquad \cos\beta = \frac{\sum M_y}{M} \qquad \cos\gamma = \frac{\sum M_z}{M}$$

空间力偶系平衡的充分必要条件是 :合力偶矩矢等于零，即

$$\vec{M} = 0$$

$$\Rightarrow \boxed{\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0}$$

--称为空间力偶系的平衡方程.

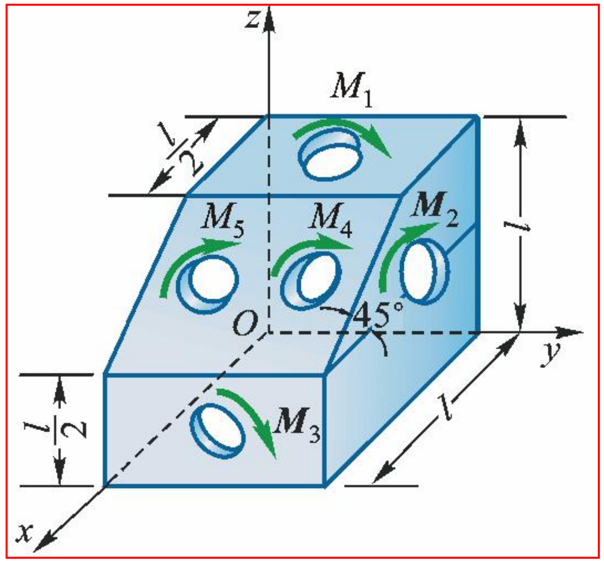
例3-5

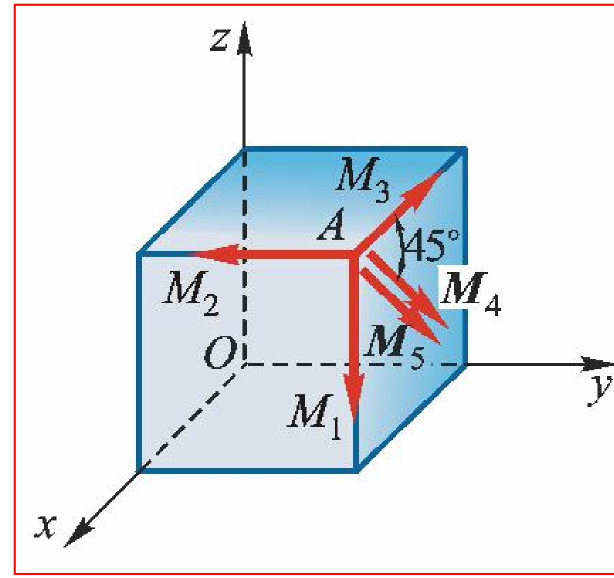
已知：在工件四个面上同时钻5个孔，每个孔所受切削力偶矩均为80N·m.

求：工件所受合力偶矩在x,y,z 轴上的投影.

解：

把力偶用力偶矩矢表示，平行移到点A .





$$M_x = \sum M_{ix} = -M_3 - M_4\cos 45^\circ - M_5\cos 45^\circ = 193.1\mathrm{N\cdot m}$$

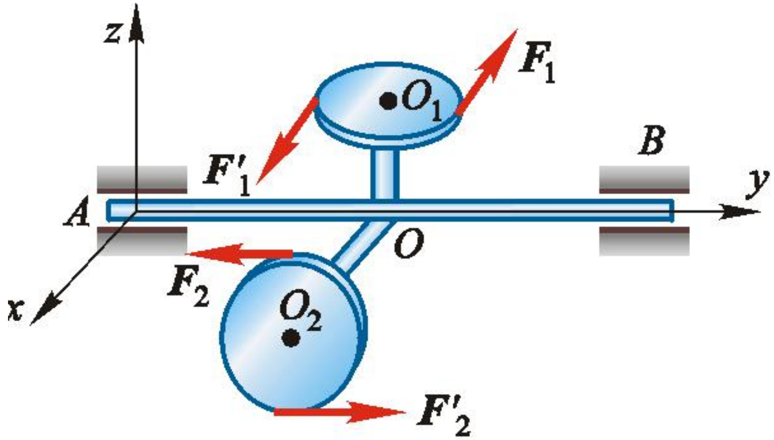
$$M_y = \sum M_{iy} = -M_2 = -80\mathbf{N\cdot m}$$

$$M_z = \sum M_{iz} = -M_1 - M_4\cos 45^\circ - M_5\cos 45^\circ = -193.1\mathbf{N\cdot m}$$

例3-6

已知：两圆盘半径均为200mm，AB =800mm，圆盘面O_1垂直于z轴，圆盘面O_2垂直于x轴，两盘面上作用有力偶，F_1=3N，F_2=5N，构件自重不计.

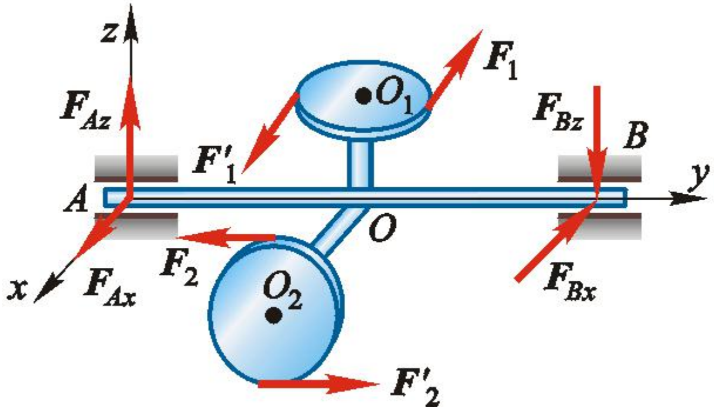
求:轴承***A*,*B***处的约束力.



解： 取整体，受力图如图所示.

$$\sum M_x = 0 \quad F_2 \cdot 400 - F_{Bz} \cdot 800 = 0$$

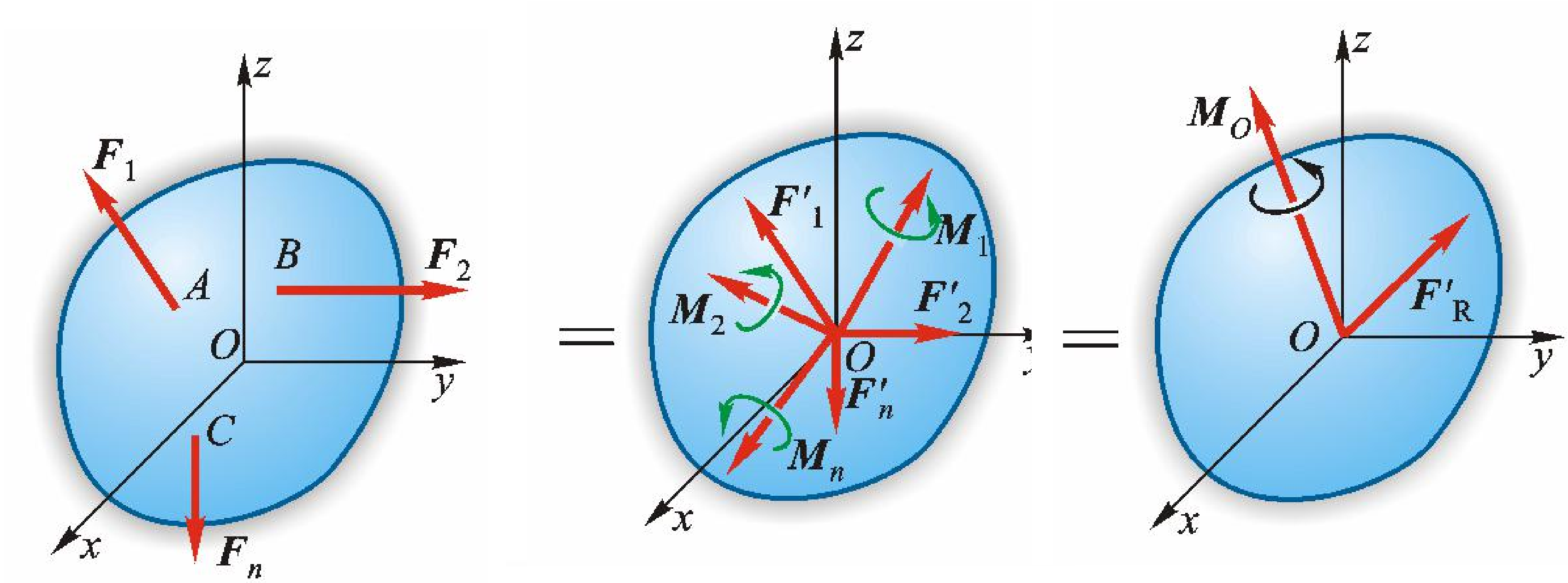
$$\sum M_z = 0 \quad F_1 \cdot 400 + F_{Bx} \cdot 800 = 0$$



$$F_{Ax} = F_{Bx} = -1.5\mathrm{N} \qquad F_{Az} = F_{Bz} = 2.5\mathrm{N}$$

§3－4 空间任意力系向一点的简化·主矢和主矩

一. 空间任意力系向一点的简化



$$\vec{F}_i' = \vec{F}_i \qquad \vec{M}_i = \vec{M}_O(\vec{F}_i)$$

空间汇交与空间力偶系等效代替一空间任意力系.

空间汇交力系的合力

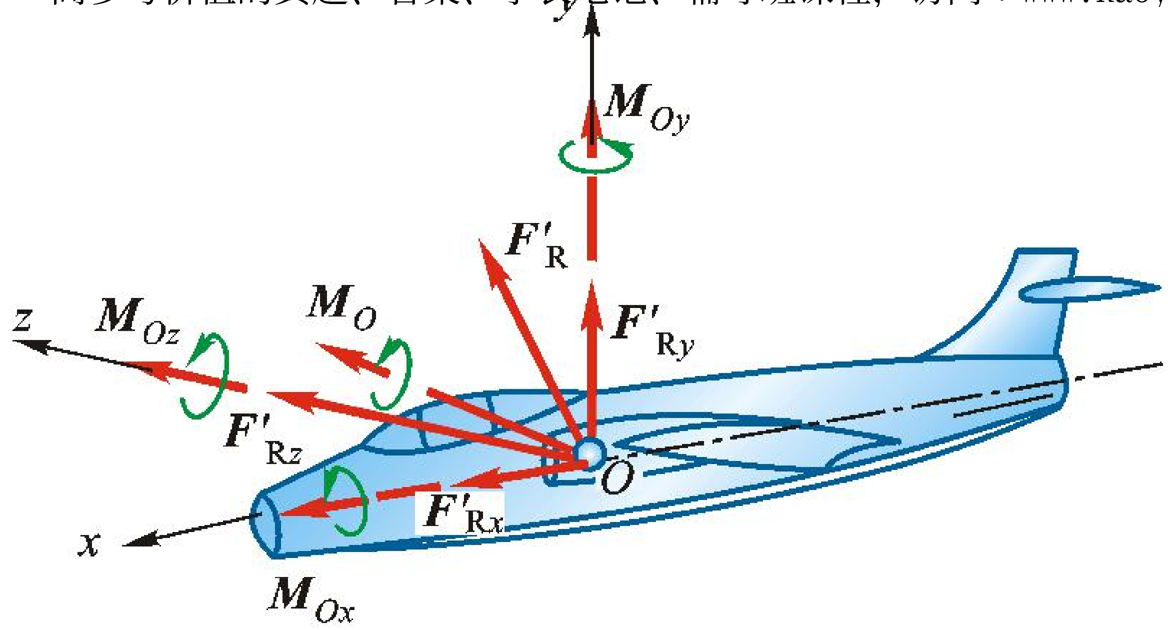
$$\vec{F}_{\mathrm{R}}' = \sum \vec{F}_i = \sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k}$$ → 主矢

空间力偶系的合力偶矩

$$\vec{M}_O = \sum \vec{M}_i = \sum \vec{M}_O(\vec{F}_i)$$ → 主矩

由力对点的矩与力对轴的矩的关系，有

$$\vec{M}_O = \sum M_x(\vec{F})\vec{i} + \sum M_y(\vec{F})\vec{j} + \sum M_z(\vec{F})\vec{k}$$



$\vec{F}'_{Rx}$	—	有效推进力	飞机向前飞行
$\vec{F}'_{Ry}$	—	有效升力	飞机上升
$\vec{F}'_{Rz}$	—	侧向力	飞机侧移
$\vec{M}_{Ox}$	—	滚转力矩	飞机绕x轴滚转
$\vec{M}_{Oy}$	—	偏航力矩	飞机转弯
$\vec{M}_{Oz}$	—	俯仰力矩	飞机仰头

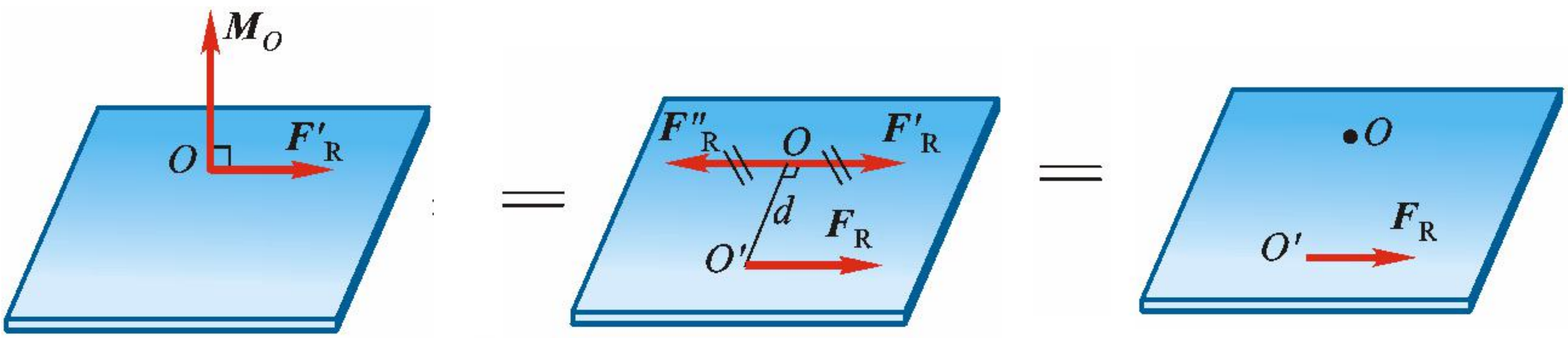
二．空间任意力系的简化结果分析（最后结果）

合力

$\vec{F}_R' \neq 0, \vec{M}_O = 0$ ⟶ 过简化中心合力

$\vec{F}_R' \neq 0, \vec{M}_O \neq 0, \vec{F}_R' \perp \vec{M}_O$ ⟶ 合力. 合力作用线距简化中心为

$$d = \left|\vec{M}_O\right| / F_R'$$



$$\vec{M}_O = \vec{d} \times \vec{F}_R = \vec{M}_O(\vec{F}_R) = \sum \vec{M}_O(\vec{F})$$

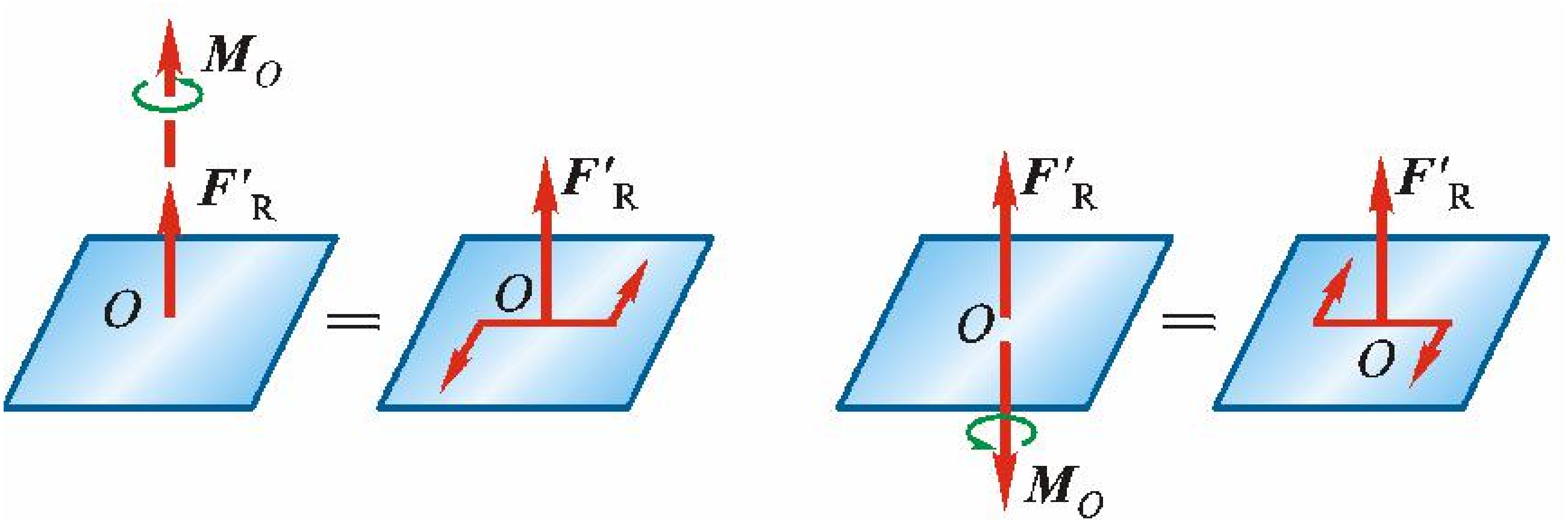
合力矩定理：合力对某点(轴）之矩等于各分力对同一点（轴）之矩的矢量和.

合力偶

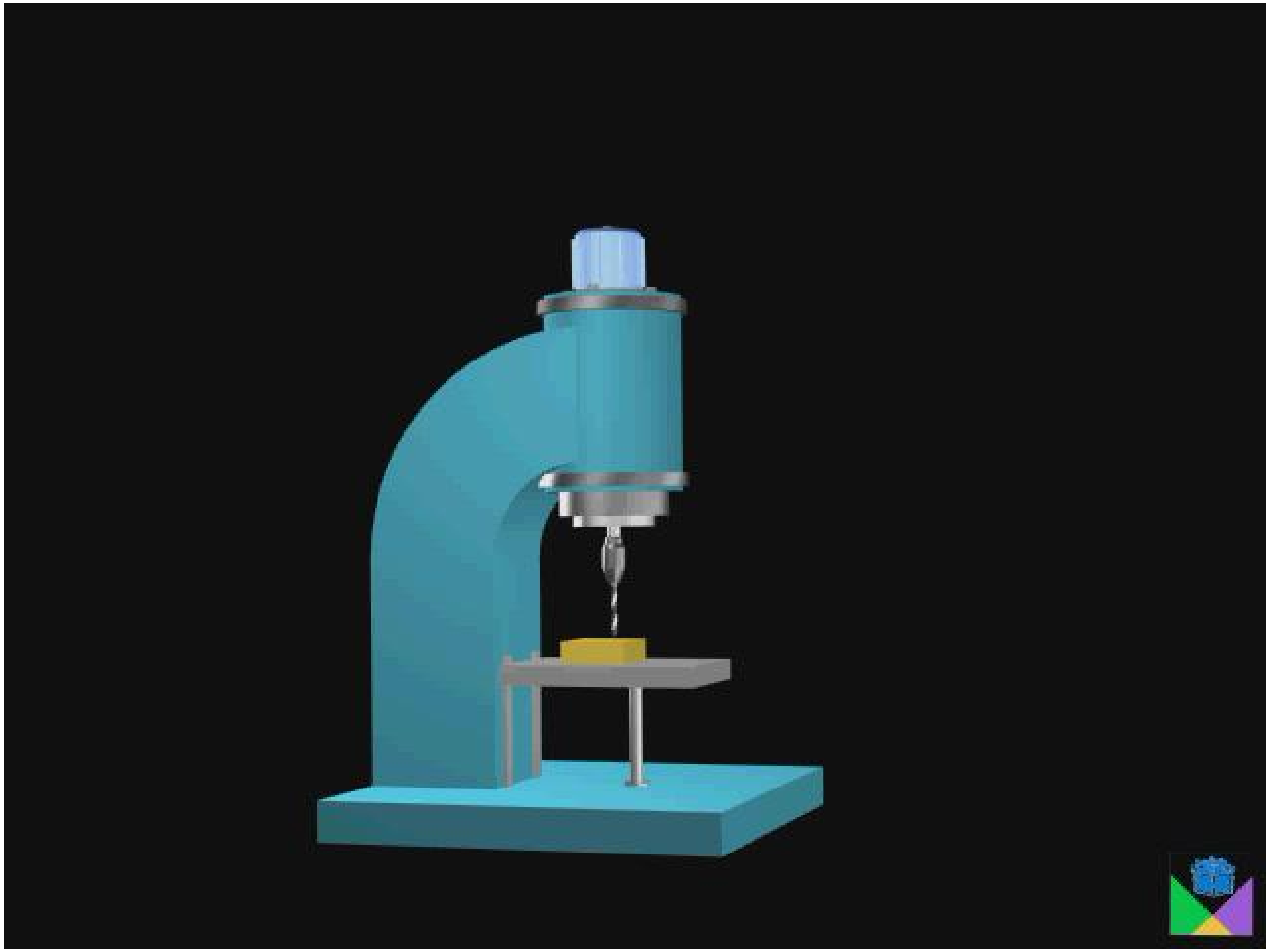
$\vec{F}_R' = 0, \vec{M}_O \neq 0$ → 一个合力偶，此时与简化中心无关。

力螺旋

$F_R' \neq 0, M_O \neq 0, F_R' // M_O$ → 中心轴过简化中心的力螺旋

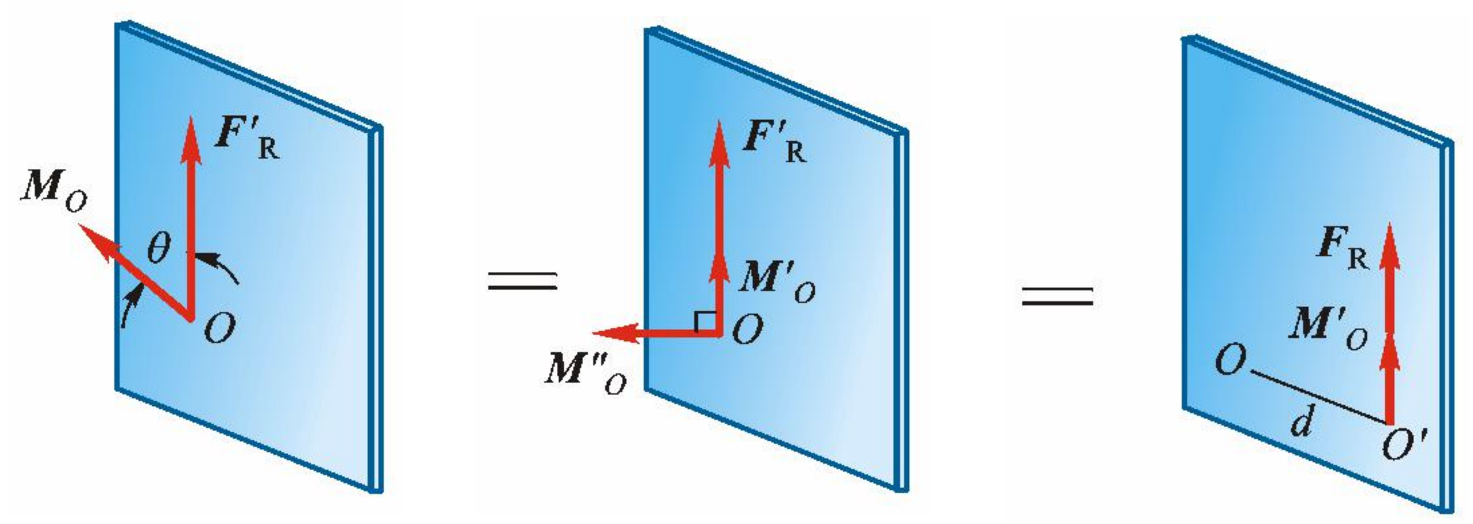


钻头钻孔时施加的力螺旋



$\vec{F}'_{\mathrm{R}} \neq 0, \vec{M}_O \neq 0, \vec{F}'_{\mathrm{R}}, \vec{M}_O$ 既不平行也不垂直

力螺旋中心轴距简化中心为 $d = \dfrac{M_O \sin\theta}{F'_{\mathrm{R}}}$



平衡

$\vec{F}'_{\mathrm{R}} = 0, \vec{M}_O = 0$ → 平衡

§3－5 空间任意力系的平衡方程

一. 空间任意力系的平衡方程

空间任意力系平衡的充要条件：

该力系的主矢、主矩分别为零.

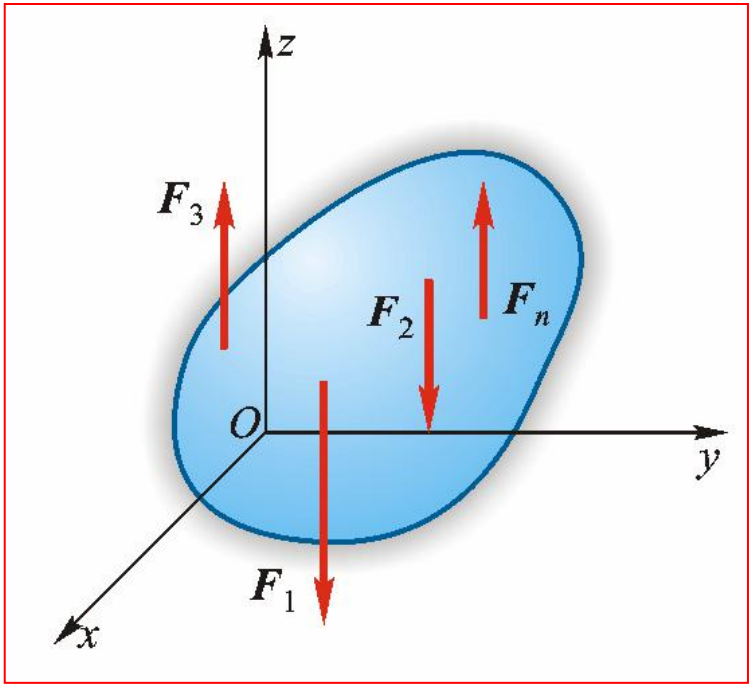
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

空间任意力系平衡的充要条件：所有各力在三个坐标轴中每一个轴上的投影的代数和等于零，以及这些力对于每一个坐标轴的矩的代数和也等于零.

二. 空间平行力系的平衡方程

$$\sum F_z = 0 \quad \sum M_x = 0 \quad \sum M_y = 0$$

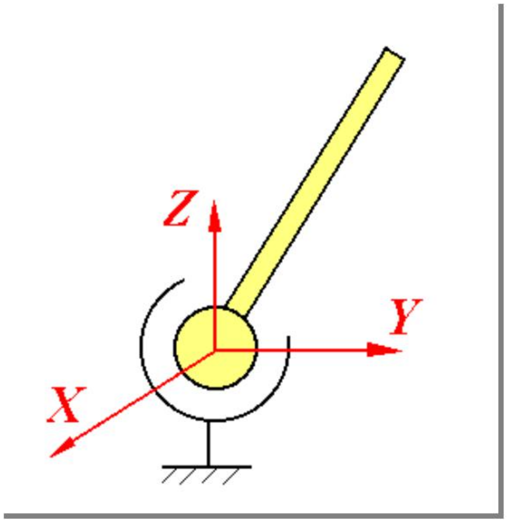


三. 空间约束类型举例

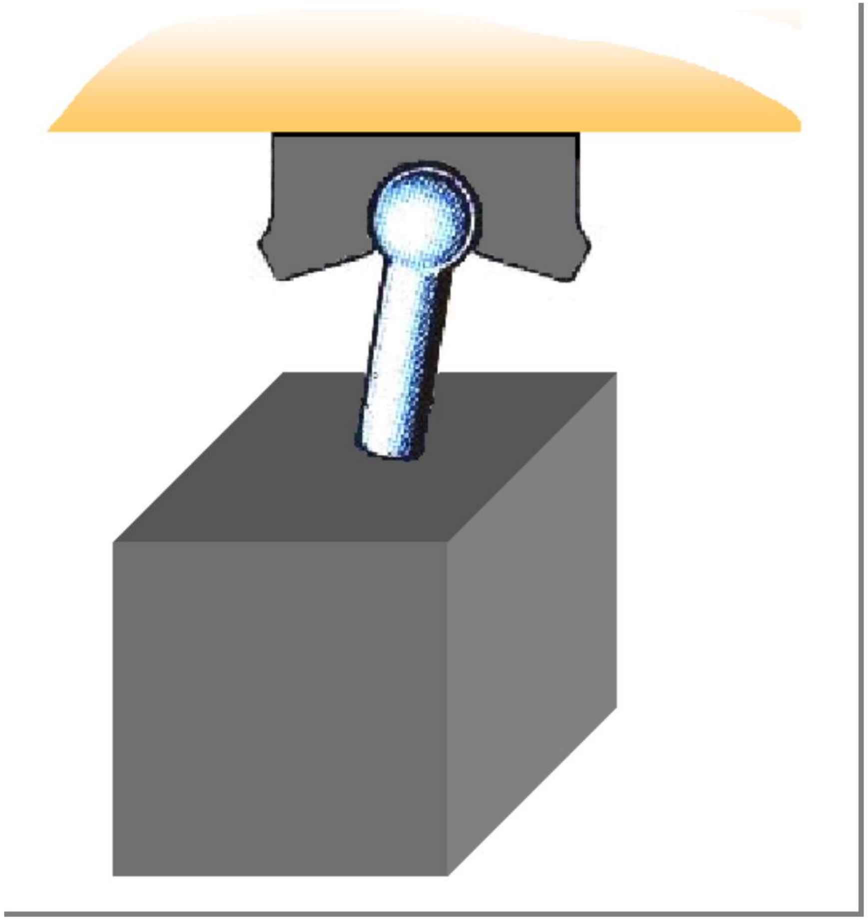
二、空间约束

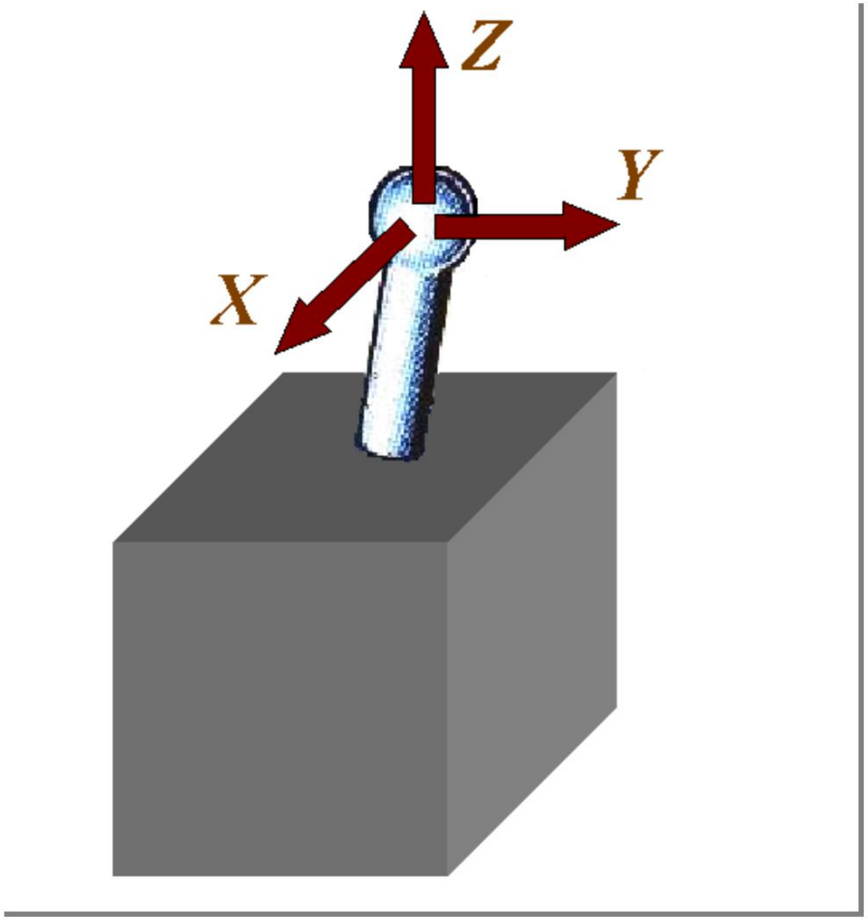
观察物体在空间的六种（沿三轴移动和绕三轴转动）可能的运动中，有哪几种运动被约束所阻碍，有阻碍就有约束反力。阻碍移动为反力，阻碍转动为反力偶。[例]

1、球形铰链

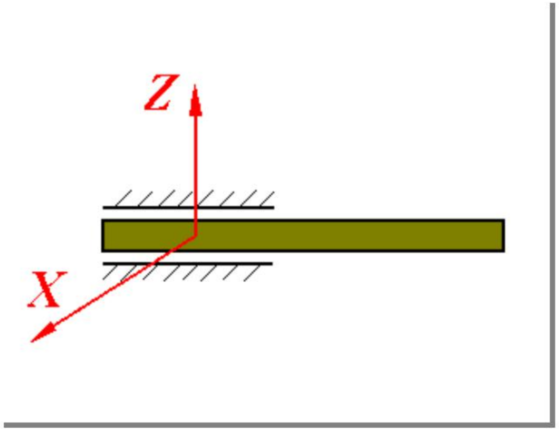


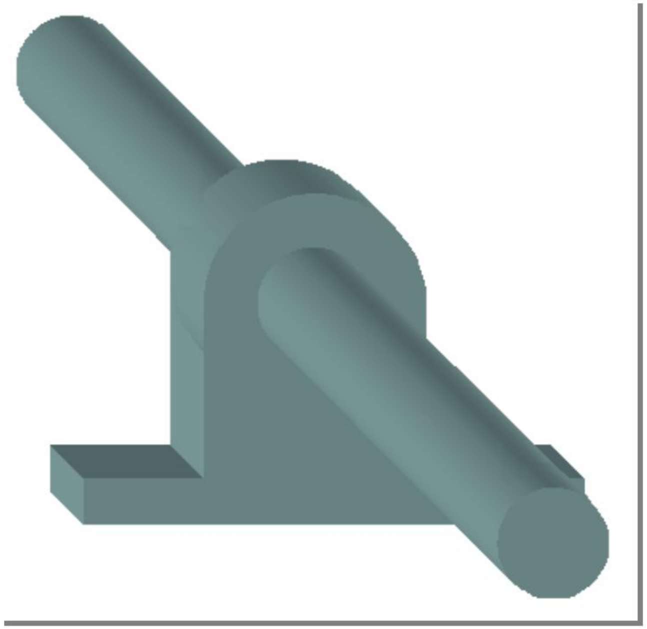
球形铰链

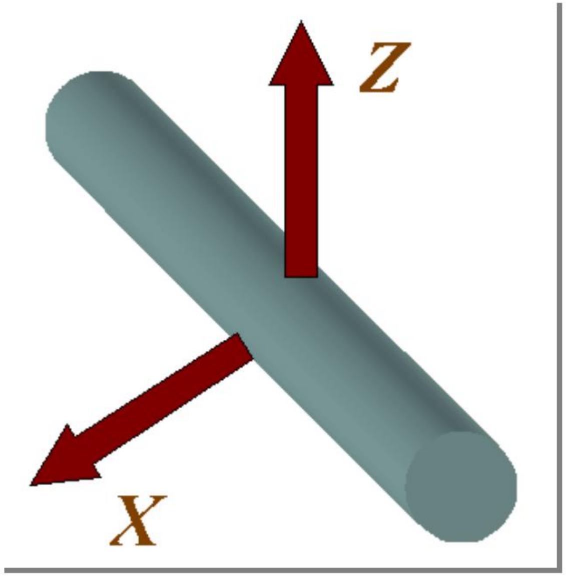


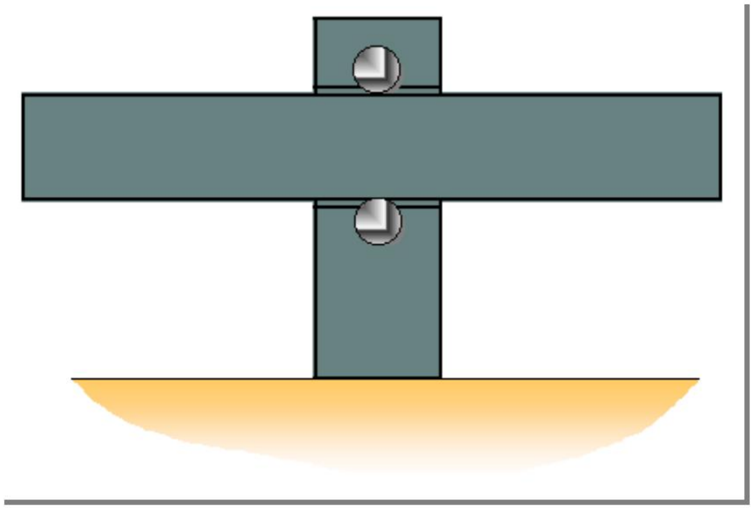


2、向心轴承，蝶铰链，滚珠(柱)轴承

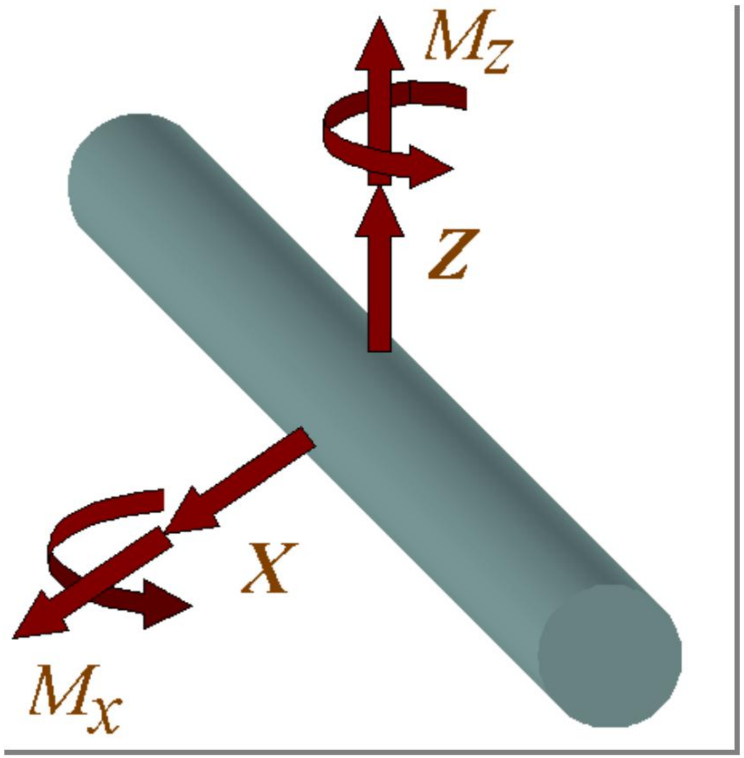


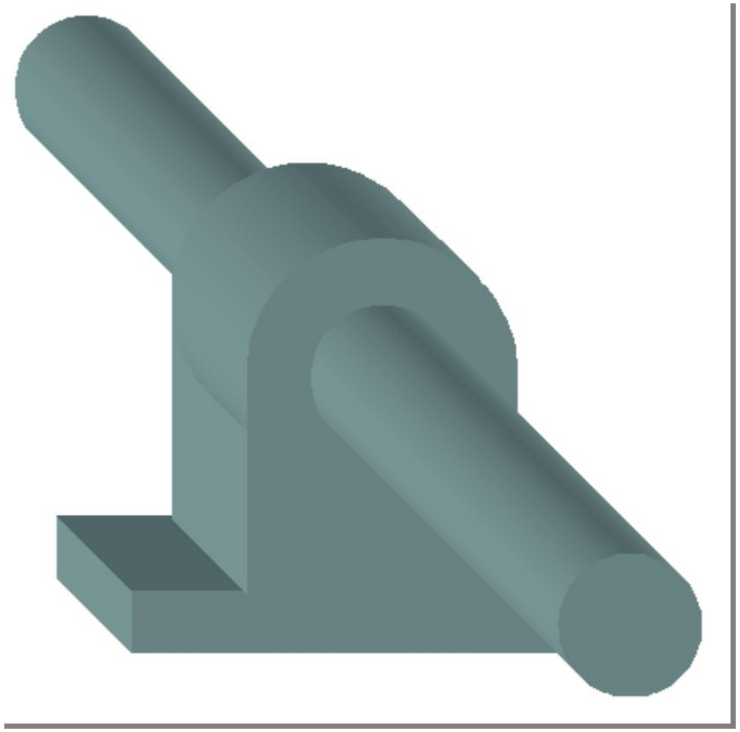


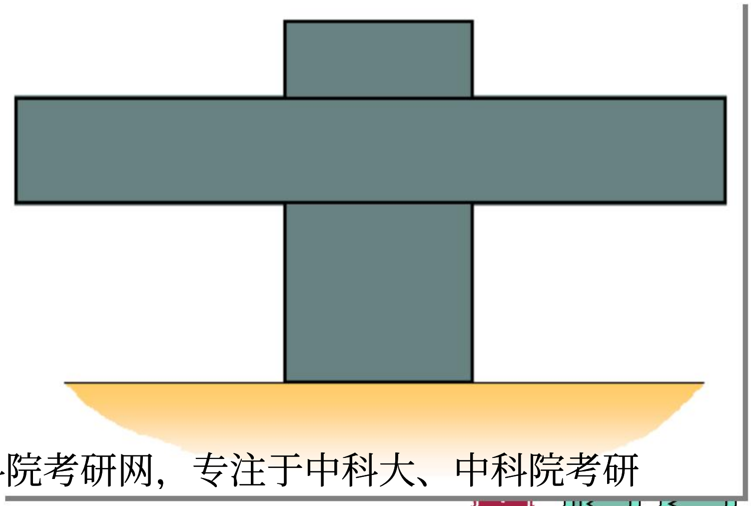




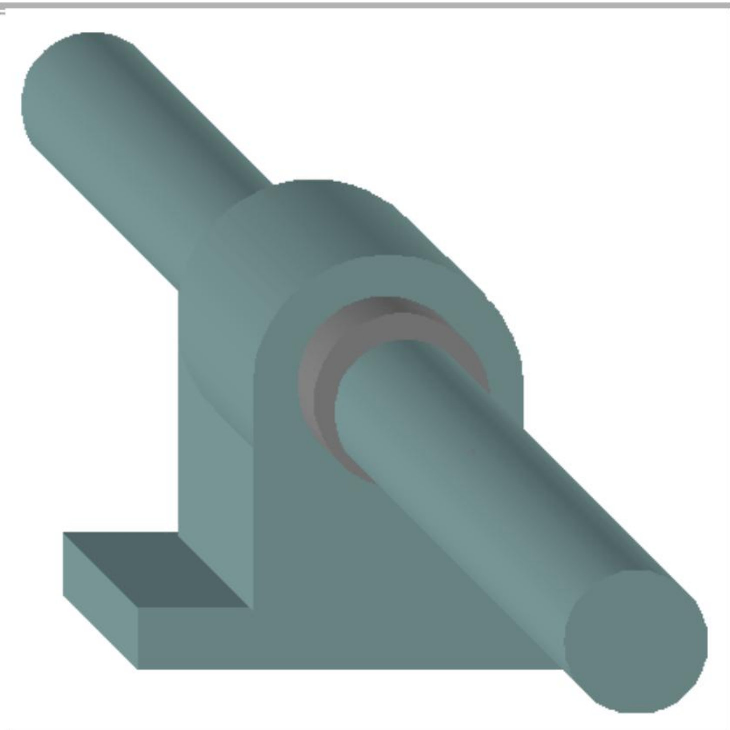
3、滑动轴承

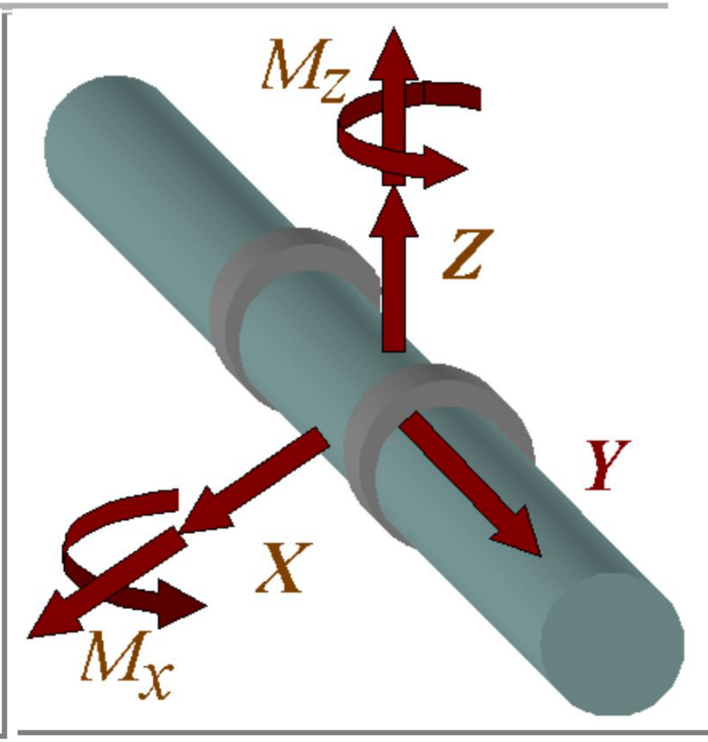


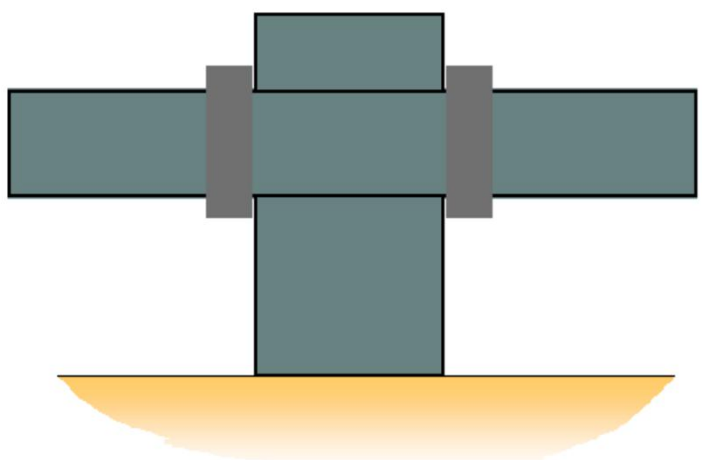


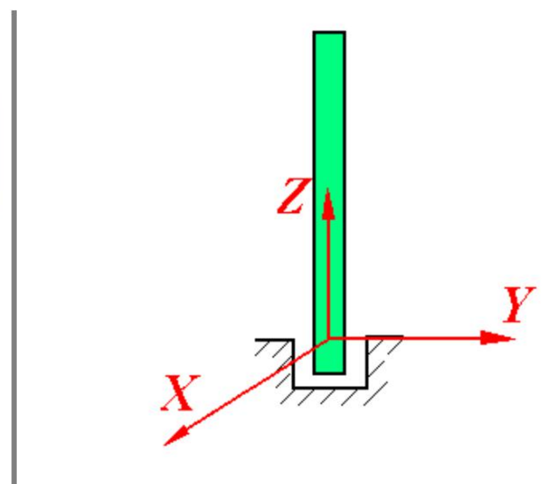


4、止推轴承

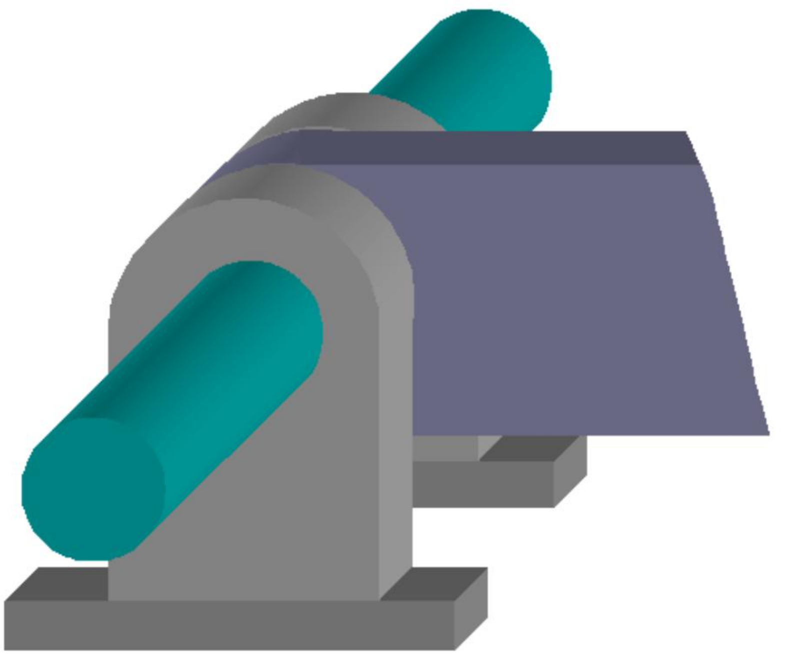


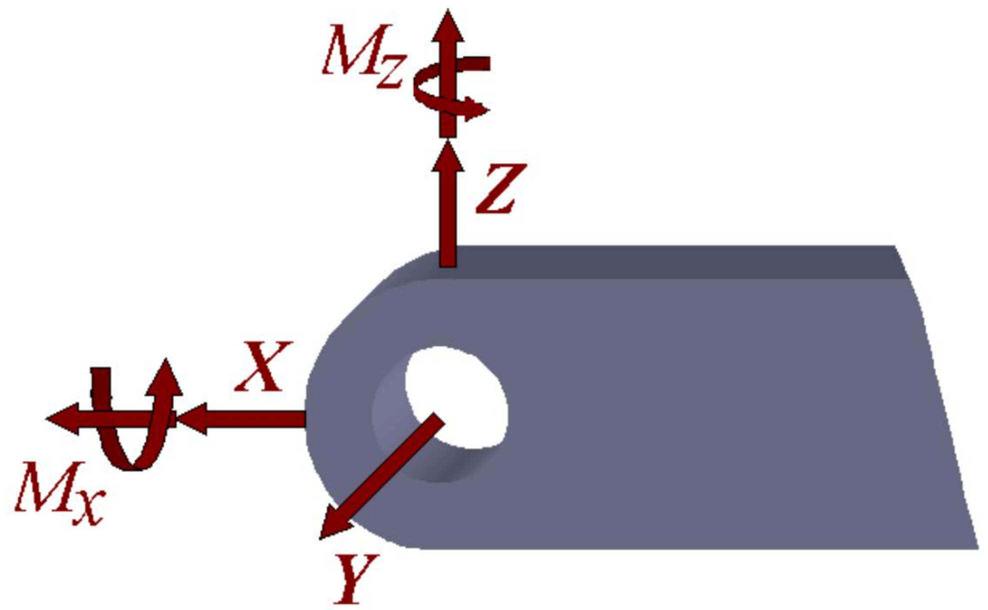


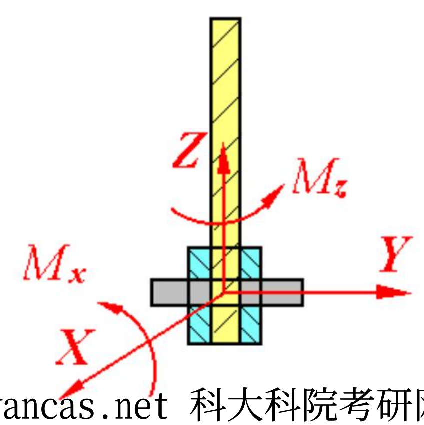




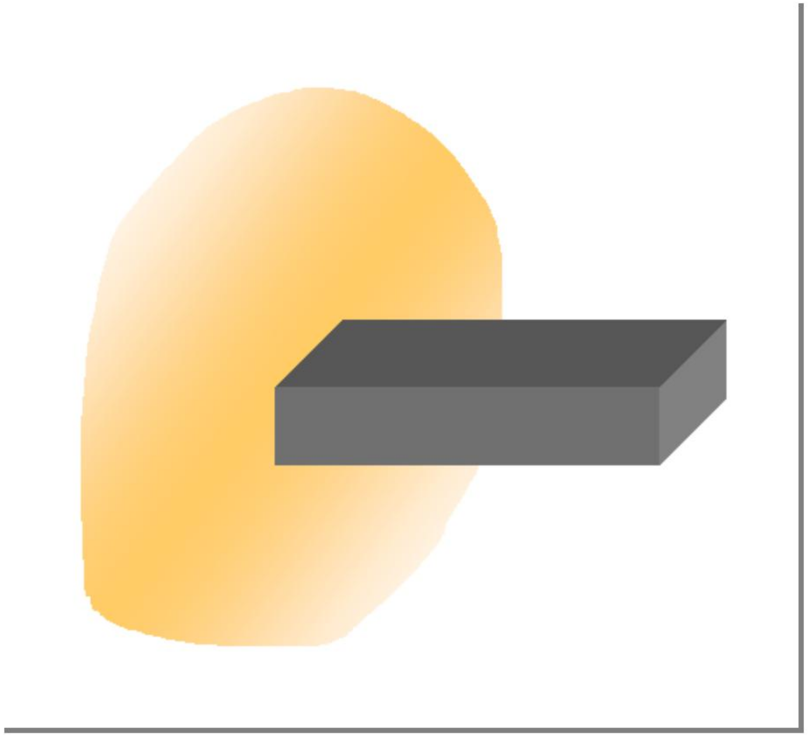
5、带有销子的夹板

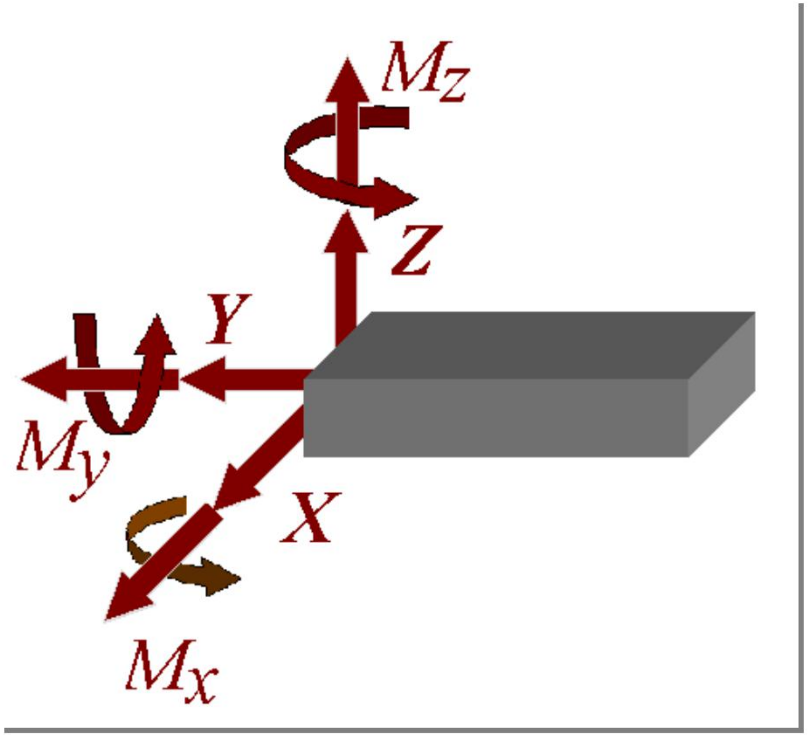


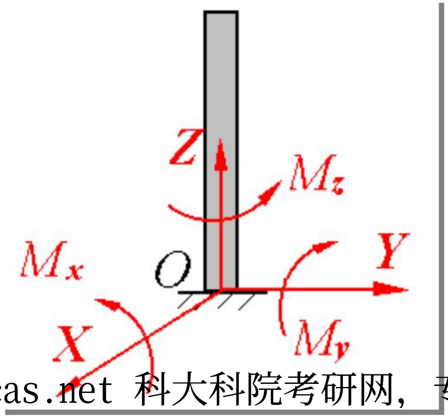




6、空间固定端

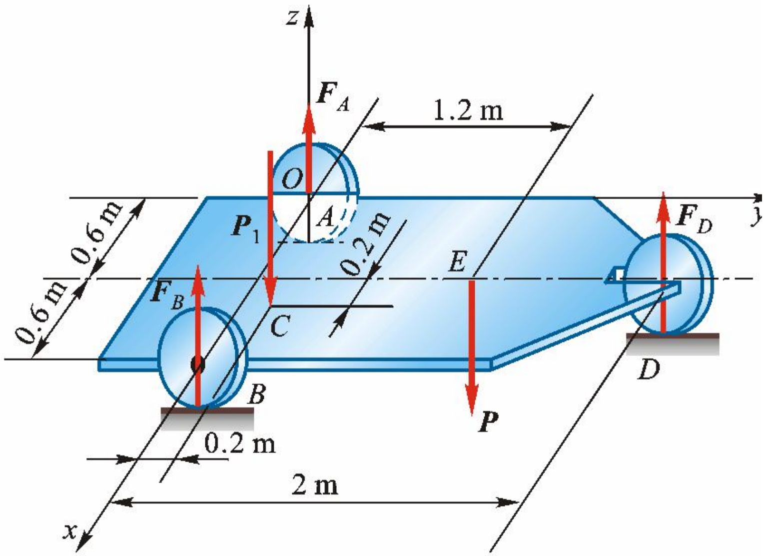






例3-8 已知：$P=8\text{kN}$，$P_1=10\text{kN}$，各尺寸如图

求：A、B、C 处约束力



解： 研究对象：小车

列平衡方程

$$\sum F_z=0 \qquad -P-P_1+F_A+F_B+F_D=0$$

$$\sum M_x(F)=0 \qquad -0.2P_1-1.2P+2F_D=0$$

$$\sum M_y(F)=0 \qquad 0.8P_1+0.6P-1.2F_B-0.6F_D=0$$

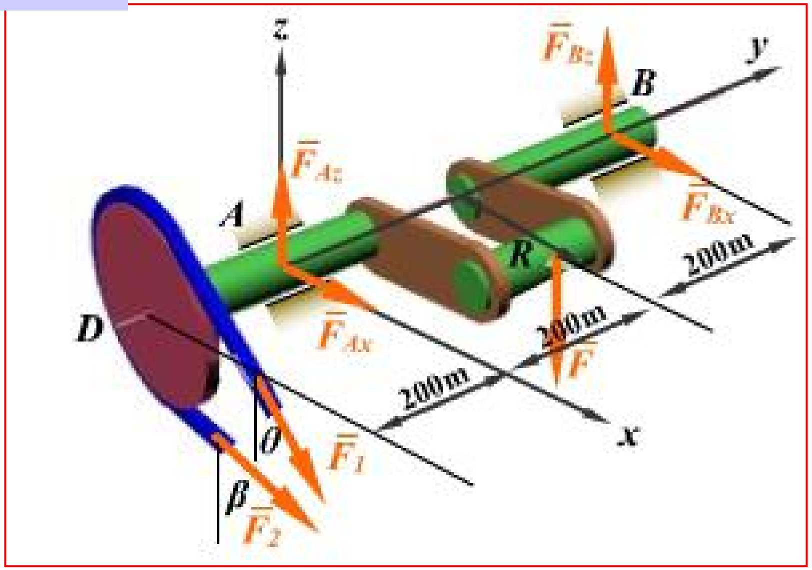
$$\Rightarrow \quad F_D=5.8\text{kN}, F_B=7.777\text{kN}, F_A=4.423\text{kN}$$

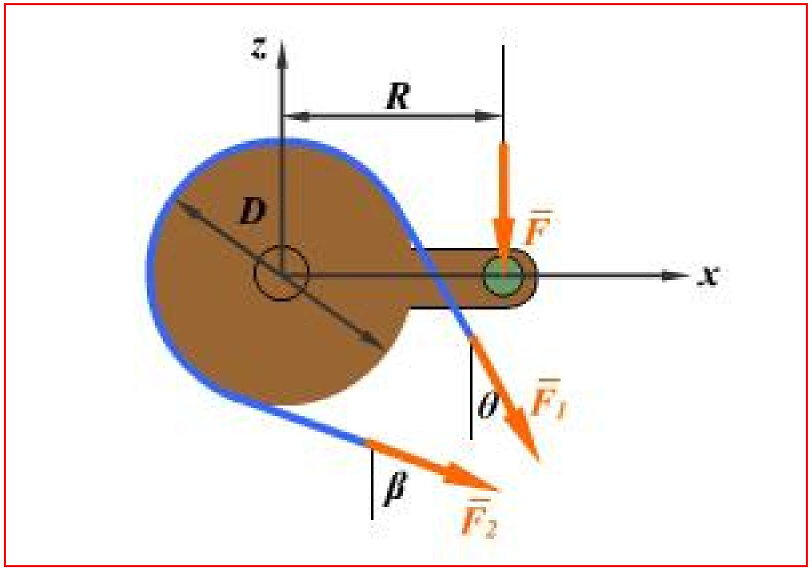
例3-9

已知：$F=2000\text{N},\ F_2=2F_1,\ \theta=30^\circ,\ \beta=60^\circ$，各尺寸如图

求：F_1, F_2 及A、B处约束力

解：研究对象，曲轴

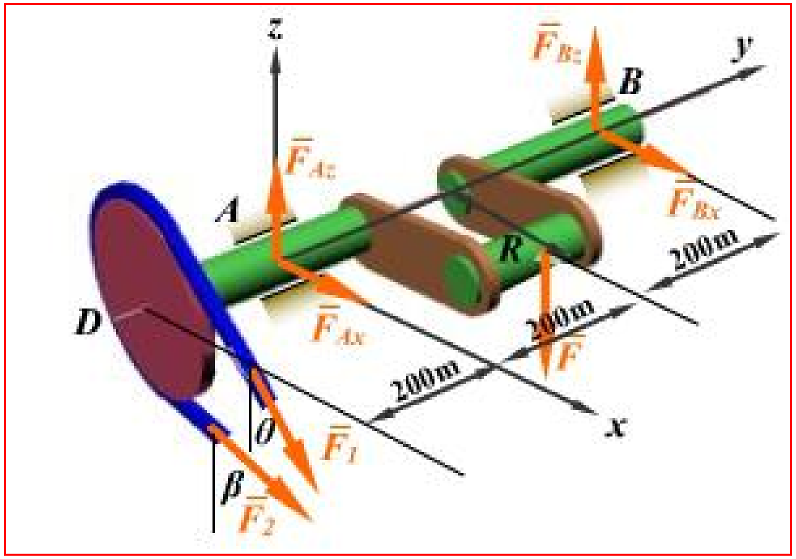


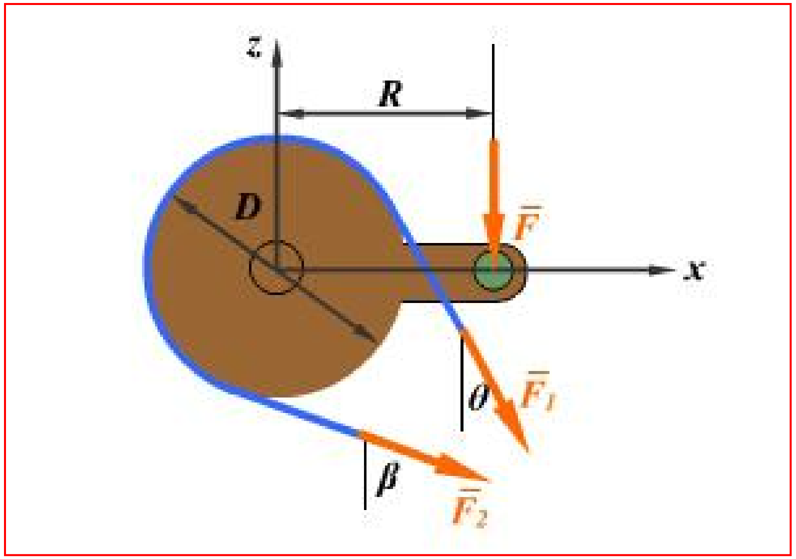


列平衡方程

$$\sum F_x=0 \qquad F_1\sin 30^\circ+F_2\sin 60^\circ+F_{Ax}+F_{Bx}=0$$

$$\sum F_y=0 \qquad 0=0$$





$$\sum F_z = 0 \qquad -F_1\cos 30^\circ - F_2\cos 60^\circ - F + F_{Az} + F_{Bz} = 0$$

$$\sum M_x(F) = 0$$

$$F_1\cos 30^\circ \times 200 + F_2\cos 60^\circ \times 200 - F \times 200 + F_{Bx} \times 400 = 0$$

$$\sum M_y(F) = 0 \qquad F\cdot R - \frac{D}{2}\times(F_2 - F_1) = 0$$

$$\sum M_z(F) = 0 \quad (F_1\sin 30^\circ + F_2\sin 60^\circ)\times 200 - F_{Bx}\times 400 = 0$$

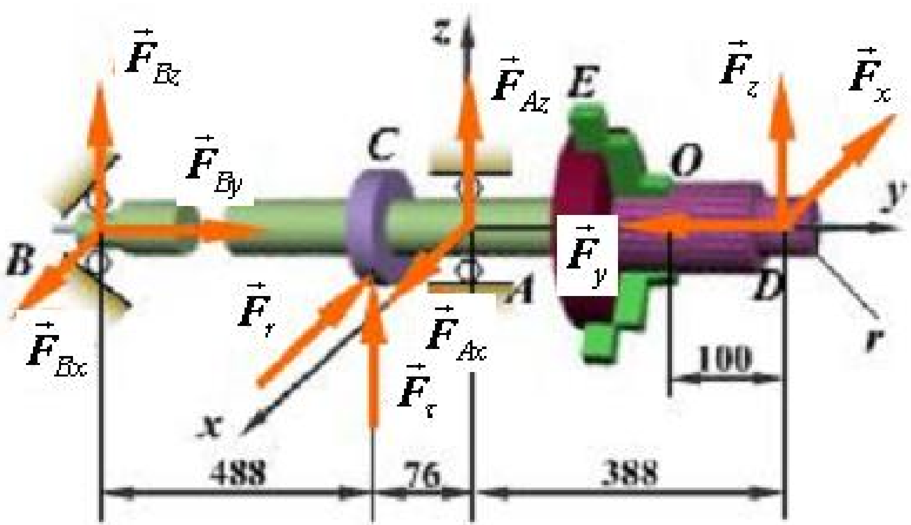
$$F_1 = 3000\text{N}, F_2 = 6000\text{N},$$

$$F_{Ax} = -1004\text{N}, F_{Az} = 9397\text{N},$$

$$F_{Bx} = 3348\text{N}, F_{Bz} = -1799\text{N},$$

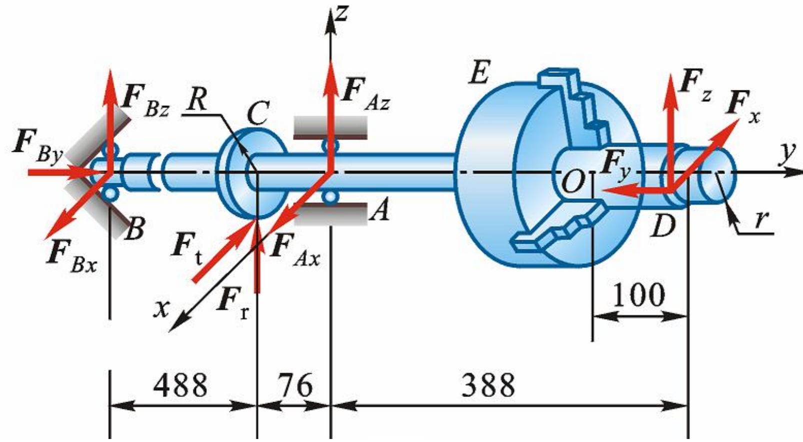
例3-10

已知： $F_x = 4.25\text{N}$， $F_y = 6.8\text{N}$， $F_z = 17\text{N}$，
$F_r = 0.36F_\tau$， $R = 50\text{mm}$， $r = 30\text{mm}$ 各尺寸如图



求： (1) $\vec{F}_r, \vec{F}_\tau$ (2) A、B处约束力 (3) O 处约束力

解：研究对象1：主轴及工件，受力图如图

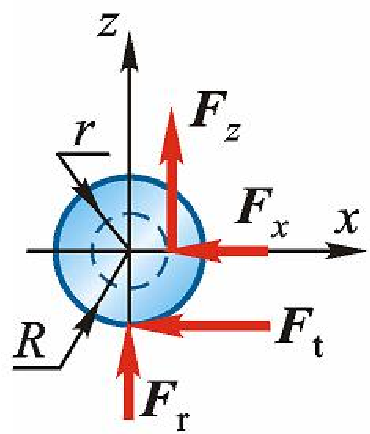


$$\sum F_x = 0 \quad -F_t + F_{Bx} + F_{Ax} - F_x = 0$$

$$\sum F_y = 0 \quad F_{By} - F_y = 0$$

$$\sum F_z = 0 \quad F_r + F_{Bz} + F_{Az} + F_z = 0$$

$$\sum M_x(F) = 0 \quad -(488+76)F_{Bz} - 76F_r + 388F_z = 0$$



$$\sum M_y(F) = 0 \quad F_t \cdot R - F_z \cdot r = 0$$

$$\sum M_z(F) = 0 \quad (488+76)F_{Bx} - 76F_t + 388F_x - 30F_y = 0$$

又： $F_r = 0.36F_t$，

$$F_t = 10.2\text{kN} \qquad F_r = 3.67\text{kN} \qquad F_{Ax} = 15.64\text{kN}$$

$$F_{Bx} = -1.19\text{kN} \qquad F_{By} = 6.8\text{kN} \qquad F_{Bz} = 11.2\text{kN}$$

研究对象2：工件受力图如图，列平衡方程

$$\sum F_x = 0 \qquad F_{Ox} - F_x = 0$$

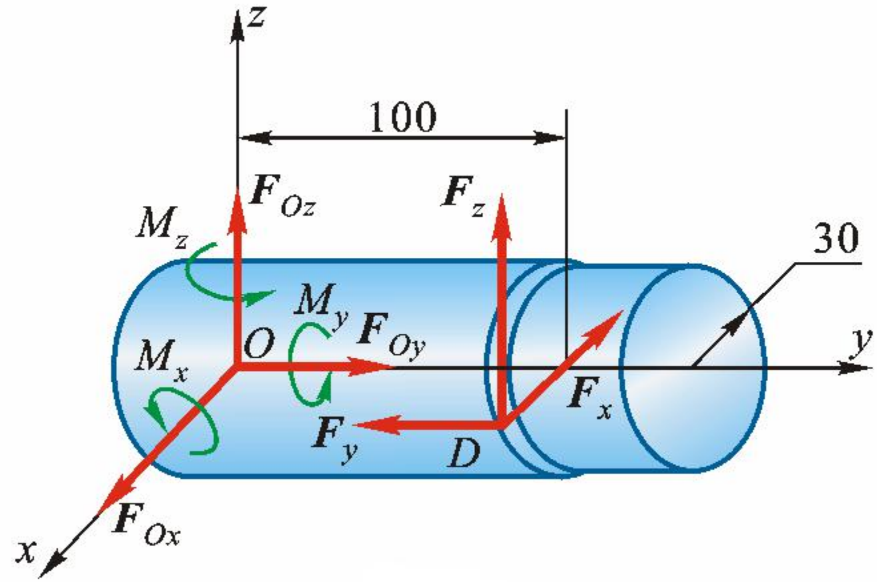
$$\sum F_y = 0 \qquad F_{Oy} - F_y = 0$$

$$\sum F_z = 0 \qquad F_{Oz} + F_z = 0$$

$$\sum M_x(F) = 0 \qquad 100F_Z + M_x = 0$$

$$\sum M_y(F) = 0 \qquad -30F_Z + M_y = 0$$

$$\sum M_z(F) = 0 \qquad 100F_x - 30F_y + M_z = 0$$

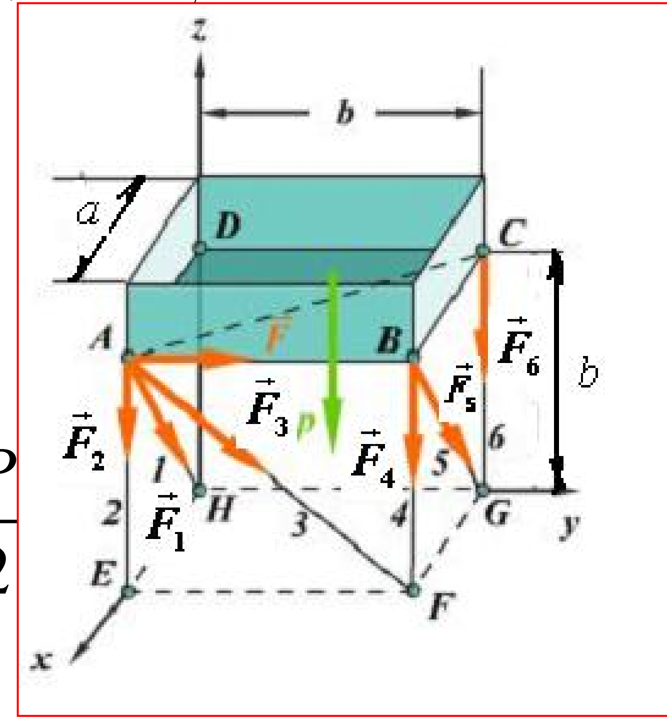


$$\Rightarrow \quad F_{Ox} = 4.25\text{kN}, F_{Oy} = 6.8\text{kN}, F_{Oz} = -17\text{kN}$$

$$M_x = -1.7\text{kN}\cdot\text{m}, M_y = 0.51\text{kN}\cdot\text{m}, M_z = -0.22\text{kN}\cdot\text{m}$$

例3-11　已知：F、P及各尺寸

求：　杆内力



解： 研究对象，长方板，列平衡方程

$$\sum M_{AB}\left(\vec{F}\right)=0 \quad -F_6 \cdot a - \frac{a}{2}\cdot P = 0 \quad F_6 = -\frac{P}{2}$$

$$\sum M_{AE}\left(\vec{F}\right)=0 \qquad F_5 = 0$$

$$\sum M_{AC}\left(\vec{F}\right)=0 \qquad F_4 = 0$$

$$\sum M_{EF}\left(\vec{F}\right)=0 \quad -F_6 \cdot a - \frac{a}{2}\cdot P - F_1 \cdot \frac{ab}{\sqrt{a^2+b^2}} = 0 \qquad F_1 = 0$$

$$\sum M_{FG}\left(\vec{F}\right)=0 \quad Fb - \frac{b}{2}\cdot P - F_2 b = 0 \qquad F_2 = 1.5P$$

$$\sum M_{BC}\left(\vec{F}\right)=0 \quad -F_2 \cdot b - \frac{b}{2}\cdot P - F_3 \cdot \cos 45^\circ \cdot b = 0 \qquad F_3 = -2\sqrt{2}P$$

§3-6 重 心

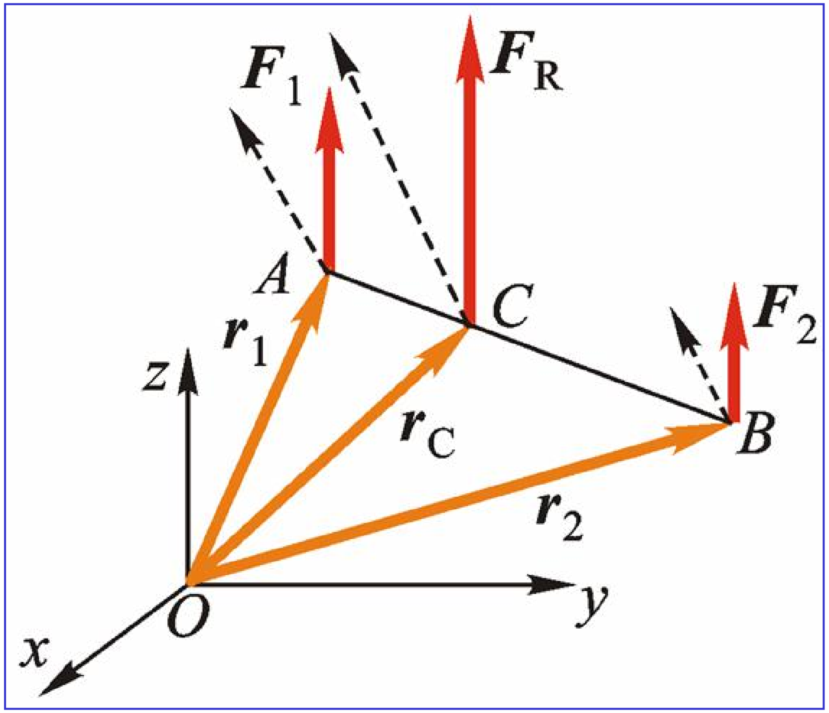
一. 平行力系中心

平行力系合力作用点的位置仅与各平行力系的大小和作用位置有关，而与各平行力的方向无关。

合力矩定理 $\vec{r}_C = \dfrac{F_1\vec{r}_1 + F_2\vec{r}_2}{F_1 + F_2}$

$$\vec{r}_C = \frac{\sum F_i \vec{r}_i}{\sum F_i}$$

$$x_C = \frac{\sum F_i x_i}{\sum F_i}$$



二. 计算重心坐标的公式

$$x_C = \frac{\sum P_i x_i}{P} \qquad y_C = \frac{\sum P_i y_i}{P} \qquad z_C = \frac{\sum P_i z_i}{P}$$

对均质物体，均质板状物体，有

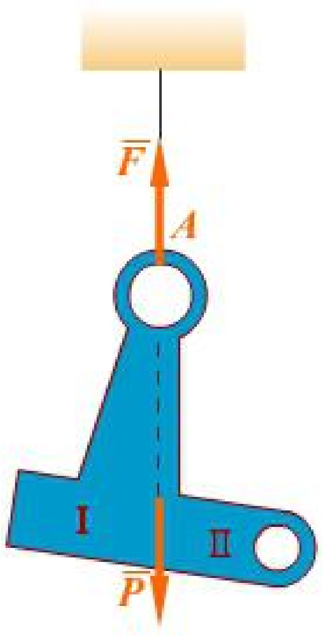
$$x_C = \frac{\sum V_i x_i}{V} \quad y_C = \frac{\sum V_i y_i}{V} \quad z_C = \frac{\sum V_i z_i}{V}$$

$$x_C = \frac{\sum A_i x_i}{A} \quad y_C = \frac{\sum A_i y_i}{A} \quad z_C = \frac{\sum A_i z_i}{A}$$

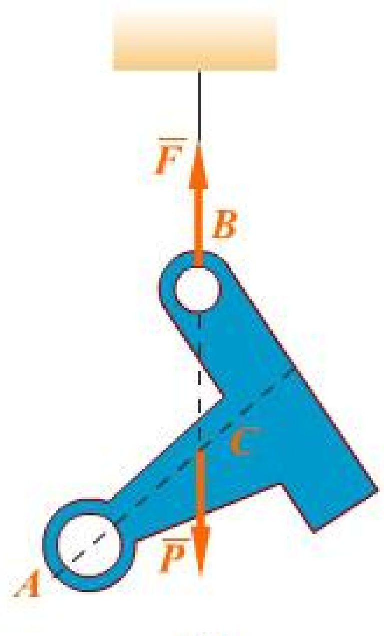
--称为重心或形心公式

三．　确定重心的悬挂法与称重法

悬挂法



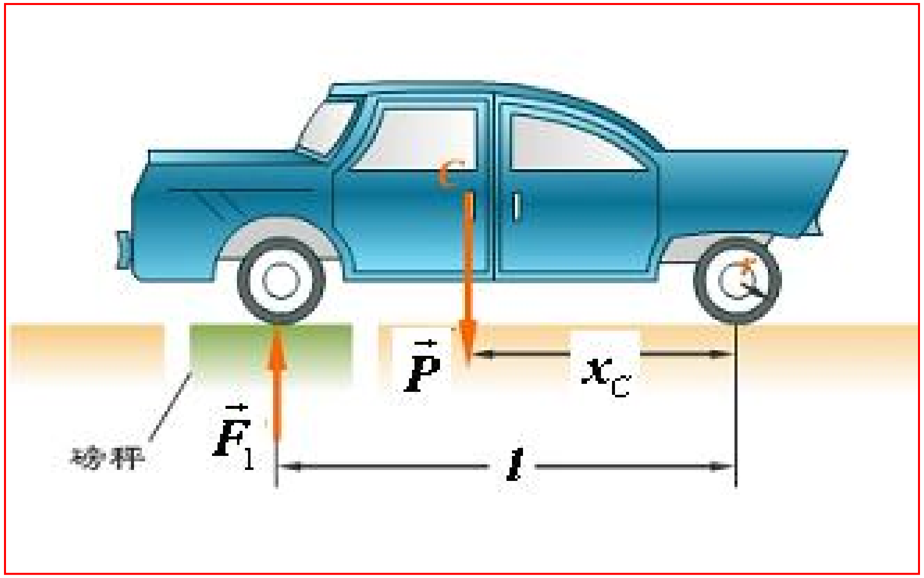
(a)



(b)

称重法

$$P\cdot x_C = F_1\cdot l \quad 则 \quad x_C=\frac{F_1}{P}l$$

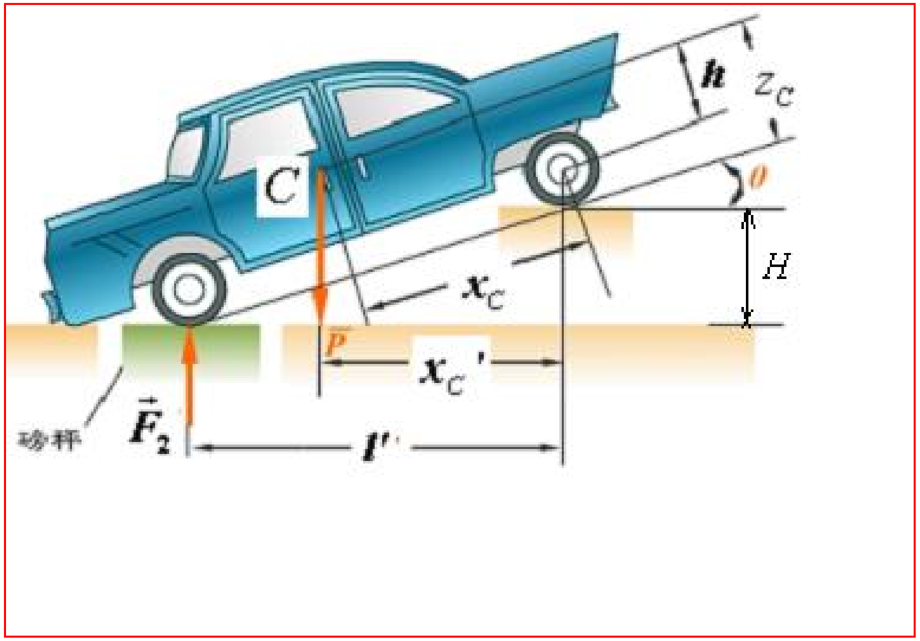


$$有\quad x_C'=\frac{F_2}{P}l'$$

$$l' = l\cos\theta$$

$$x_C' = x_C\cos\theta + h\sin\theta$$

$$\sin\theta=\frac{H}{l} \qquad \cos\theta=\frac{\sqrt{l^2-H^2}}{l}$$



$$z_C = r + \frac{F_2-F_1}{P}\cdot\frac{1}{H}\cdot\sqrt{l^2-H^2}$$

例3-12

已知：均质等厚Z字型薄板尺寸如图所示.

求：其重心坐标

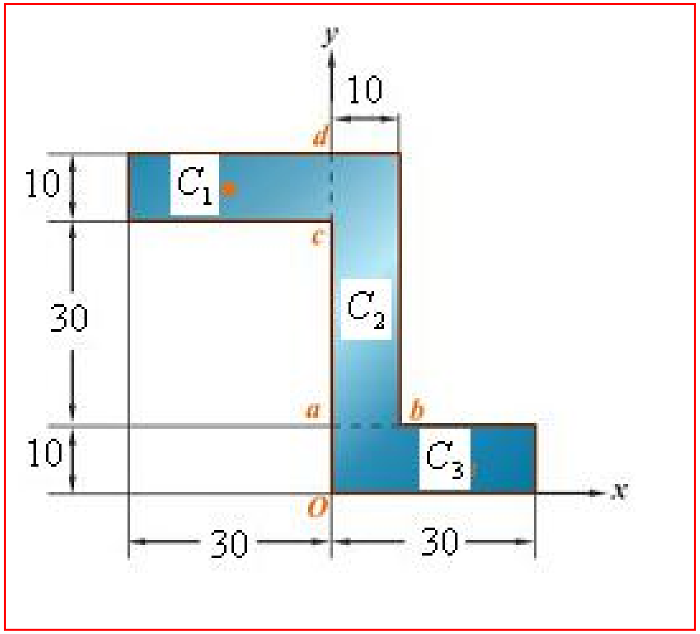
解： 厚度方向重心坐标已确定，只求重心的*x,y*坐标即可.

用虚线分割如图，为三个小矩形，其面积与坐标分别为

$$x_1 = -15\text{mm} \quad y_1 = 45\text{mm} \quad A_1 = 300\text{mm}^2$$

$$x_2 = 5\text{mm} \quad y_2 = 30\text{mm} \quad A_2 = 400\text{mm}^2$$

$$x_3 = 15\text{mm} \quad y_3 = 5\text{mm} \quad A_3 = 300\text{mm}^2$$



则

$$x_C = \sum \frac{A_i x_i}{A} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3} = 2\text{mm}$$

$$y_C = \sum \frac{A_i y_i}{A} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = 27\text{mm}$$

例3-13

已知：等厚均质偏心块的 $R=100\text{mm}, r=17\text{mm}, b=13\text{mm}$

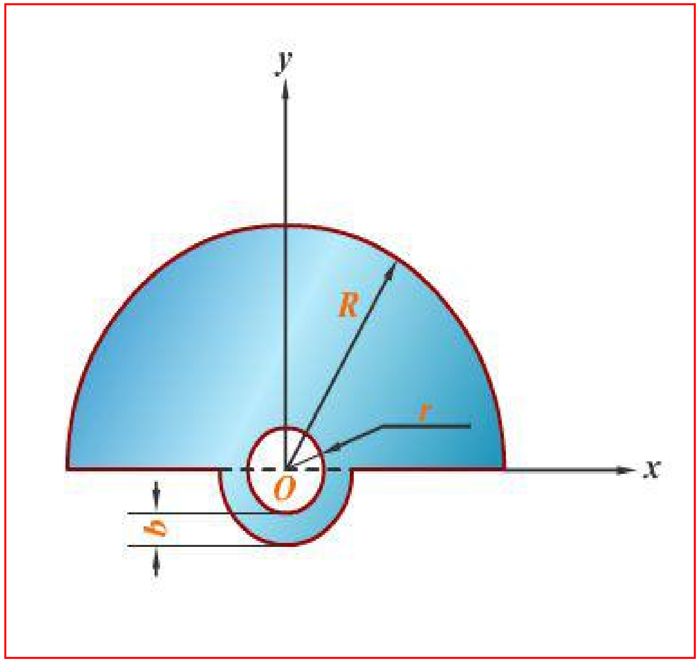
求：其重心坐标.

解： 用负面积法，为三部分组成.

由对称性，有 $x_C=0$

$$A_1=\frac{\pi}{2}R^2, A_2=\frac{\pi}{2}(r+b)^2, A_3=-\pi r^2$$

$$y_1=\frac{4R}{3\pi}, y_2=-\frac{4(r+b)}{3\pi}, y_3=0$$



由 $y_C=\dfrac{\sum A_i y_i}{A}$

得 $y_C=\dfrac{A_1 y_1+A_2 y_2+A_3 y_3}{A_1+A_2+A_3}=40.01\text{mm}$

习题

P105　3-3, 14, 17, 19, 22, 25, 26