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Mechanisms and Machine Theory
(机械原理)

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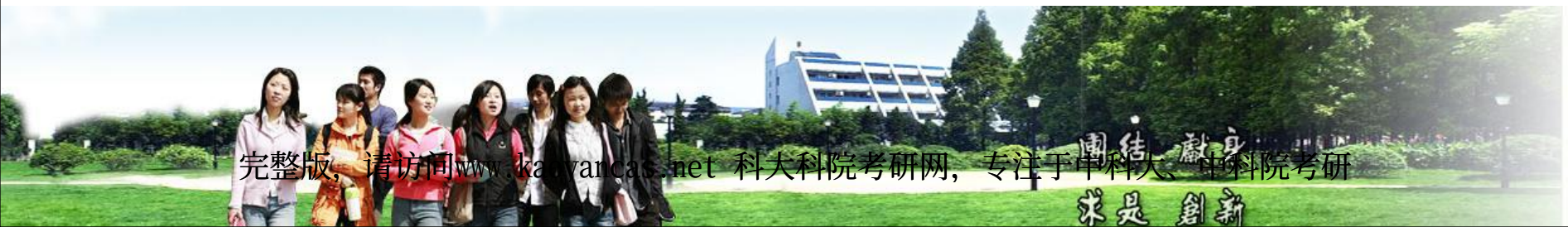

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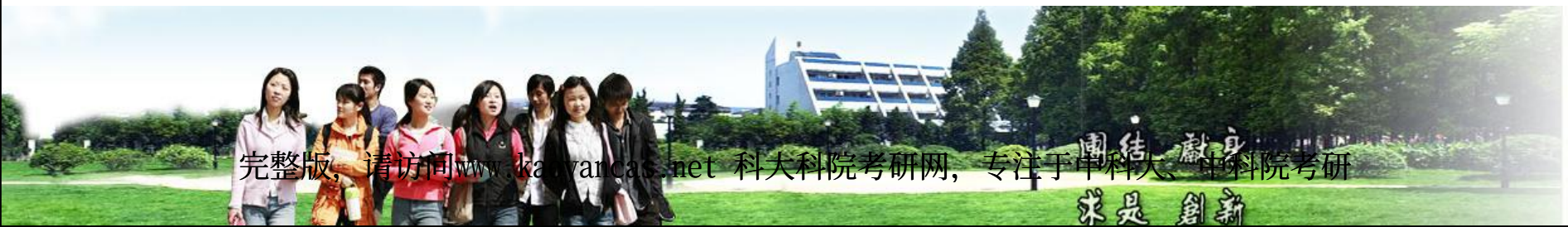
Chapter 2
Structural Analysis(结构分析)
of
Planar Mechanisms(平面机构)



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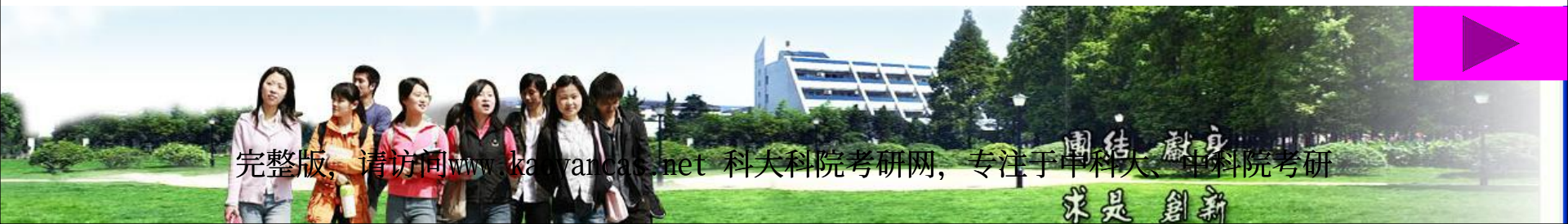
Planar mechanism平面机构：

All links of a mechanism move in planes that remains parallel to each other各构件的相对运动平面互相平行（常用的机构大多数为平面机构）。

Spatial mechanism空间机构：

至少有两个构件能在三维空间中相对运动。



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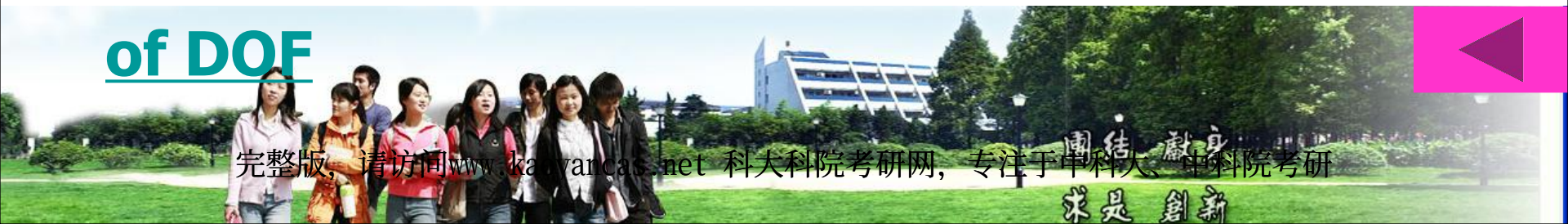
2.1 Purpose of Structural Analysis

2.2 Planar Kinematic Pairs(平面运动副) and Planar Mechanisms

2.3 The Kinematic Diagram of a Mechanism(机构运动简图)

2.4 Degree of Freedom of a Mechanism

2.5 Points for Attention during the Calculation of DOF



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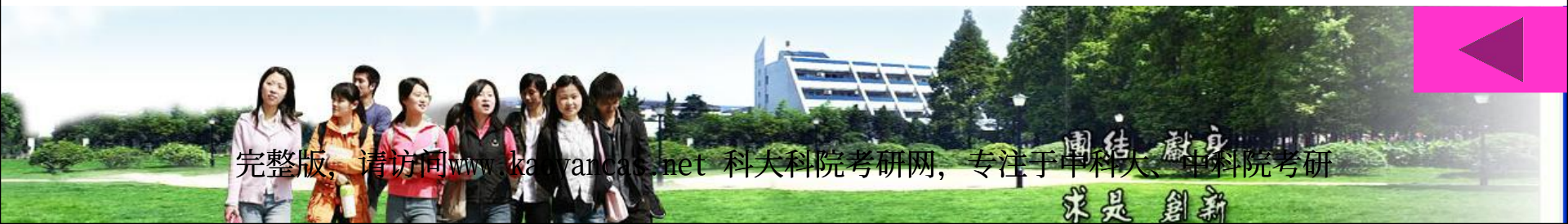
2.1 Purpose of Structural Analysis

机构结构分析的目的

- 探讨机构具有确定运动的条件
- 机构的分类
- 画机构的运动简图



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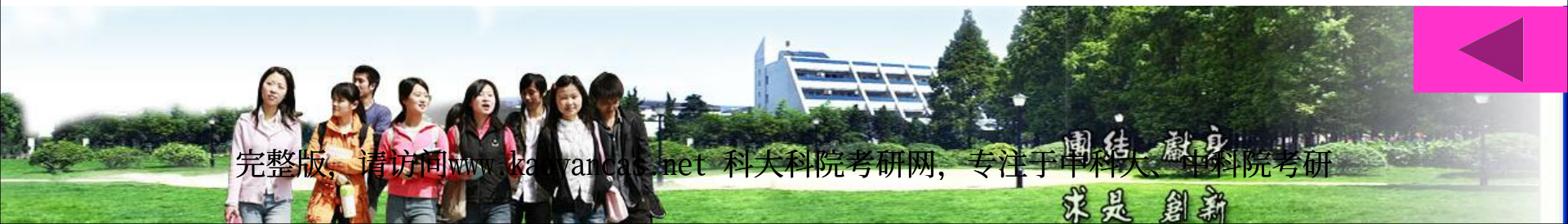
2.2 Planar Kinematic Pairs(平面运动副) and Planar Mechanisms

2.2.1 Kinematic Pairs

2.2.2. Kinematic Chain(运动链) and Mechanism



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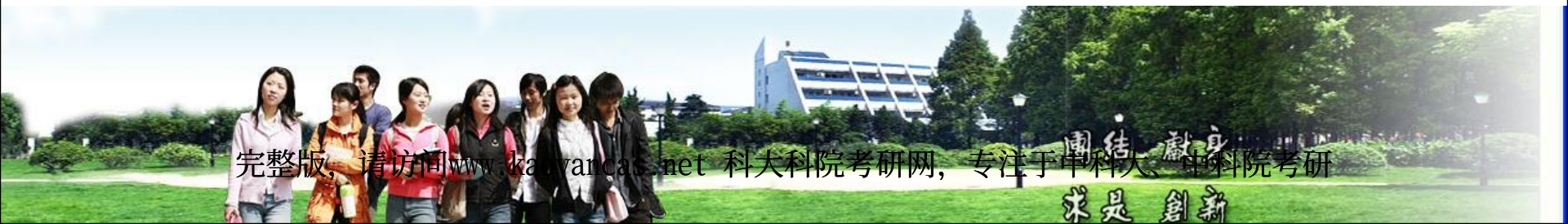
2.2.1 Kinematic Pairs

机构的组成要素 { Link构件

Kinematic Pairs运动副

In order to transmit(传输) and transform(转换) motion, every link must be kept permanently(永久地) in contact(接触) with other links by some kind of connection and have motion relative to them. Such a mobile connection(可动连接) is called a kinematic pair.



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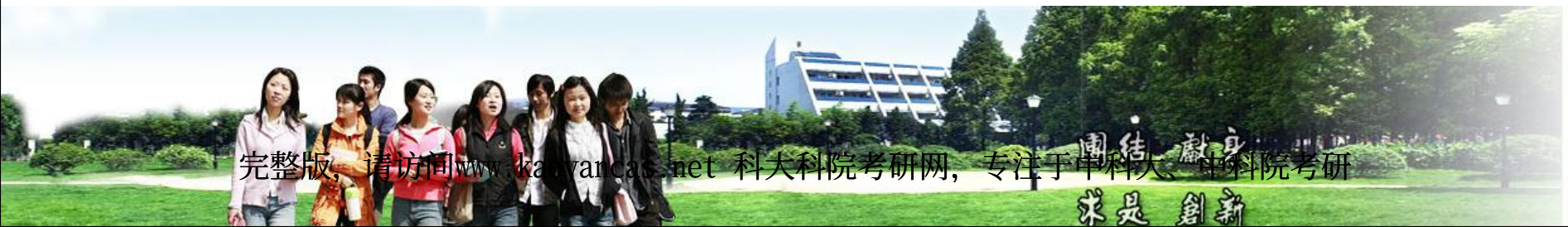
Types of kinematic pairs:
（根据两构件接触方式的不同）

面接触
lower pair

- **Revolute pair(转动副)**
- **Sliding pair or Prismatic pair(移动副)**
- **Screw pair(螺旋副)**
- **Spherical pair(球面副)**

点、线接触
higher pair

- **Gear pair(齿轮副)**
- **Cam pair(凸轮副)**



面接触——lower pair低副

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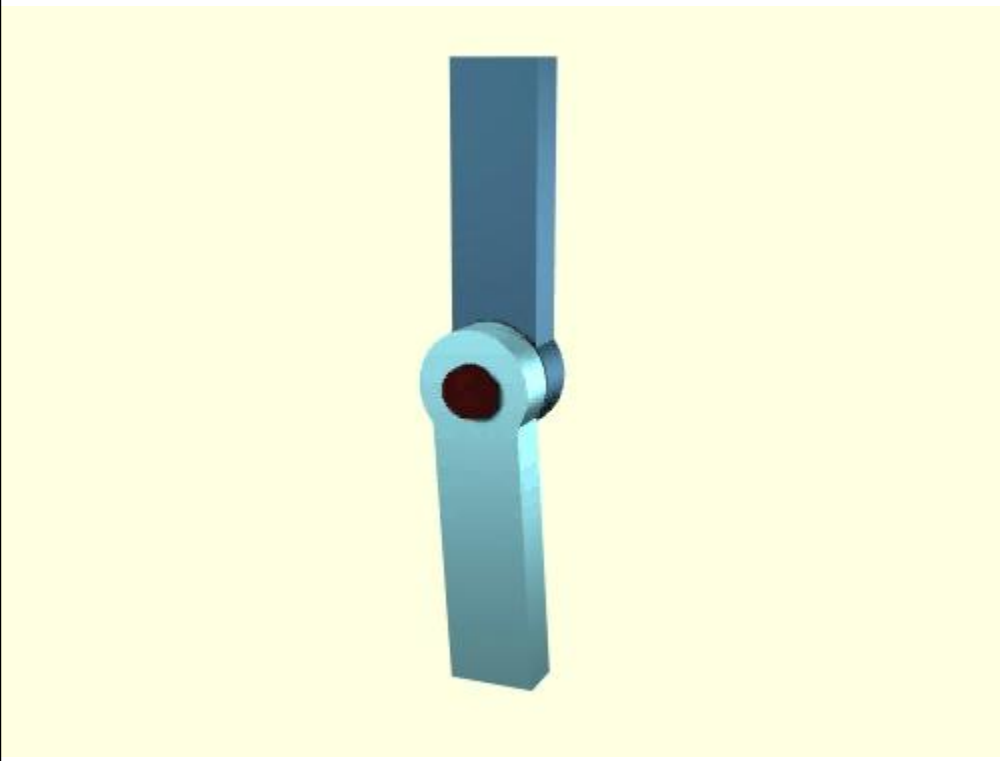
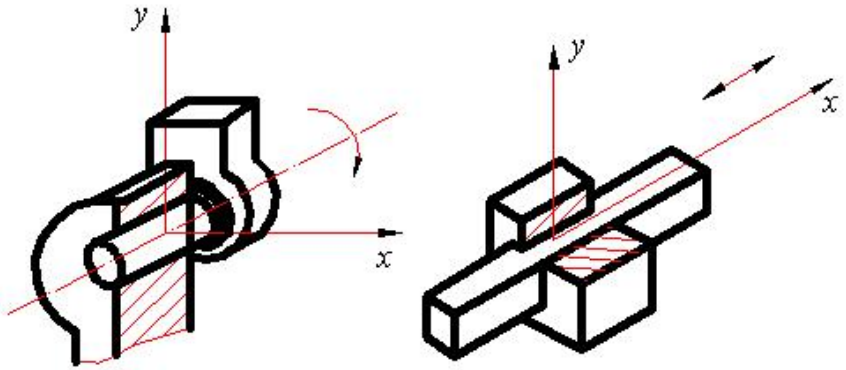
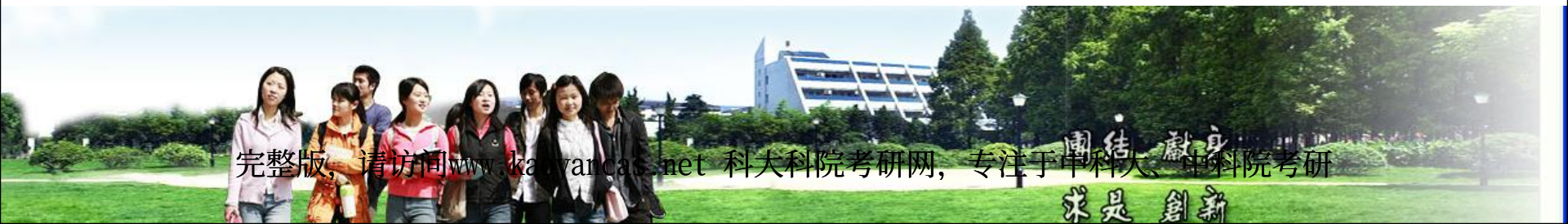
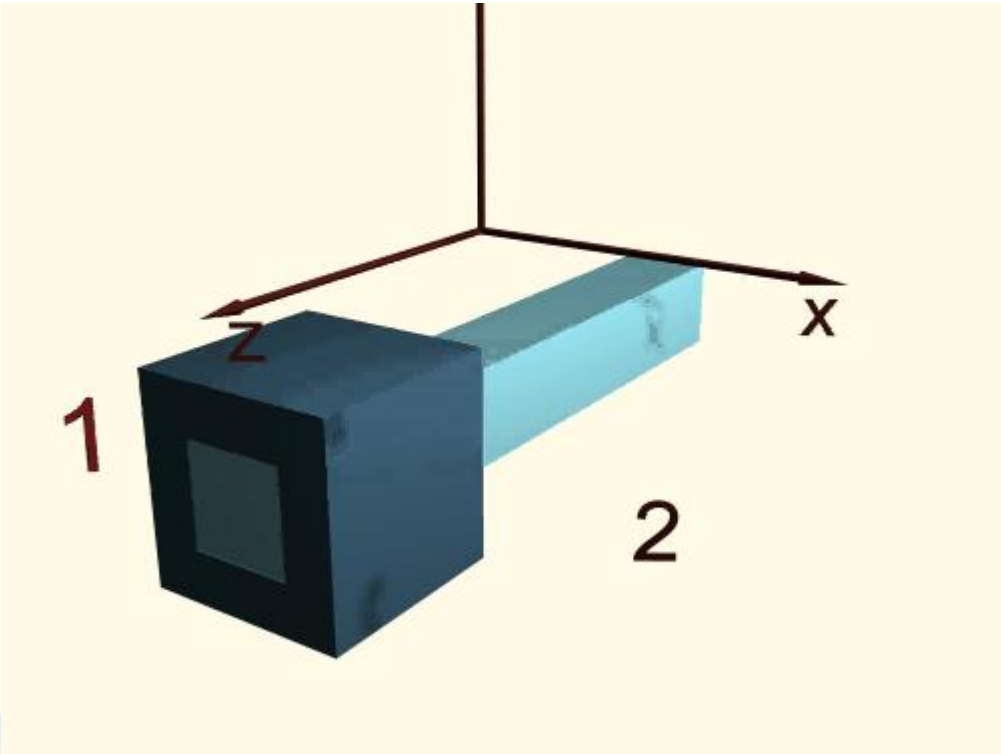




A pair that permits only relative rotation is called a revolute pair (转动副).



A pair that allows only relative rectilineal(直线的) translation(平移) is called a sliding pair or prismatic pair(移动副).



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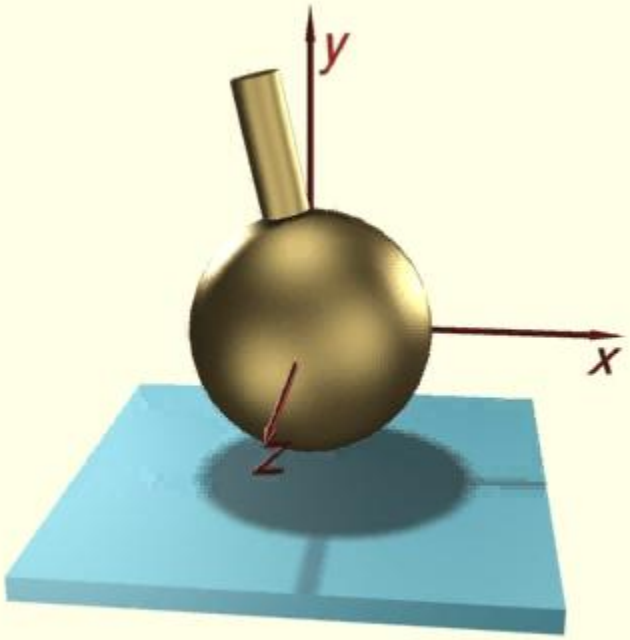
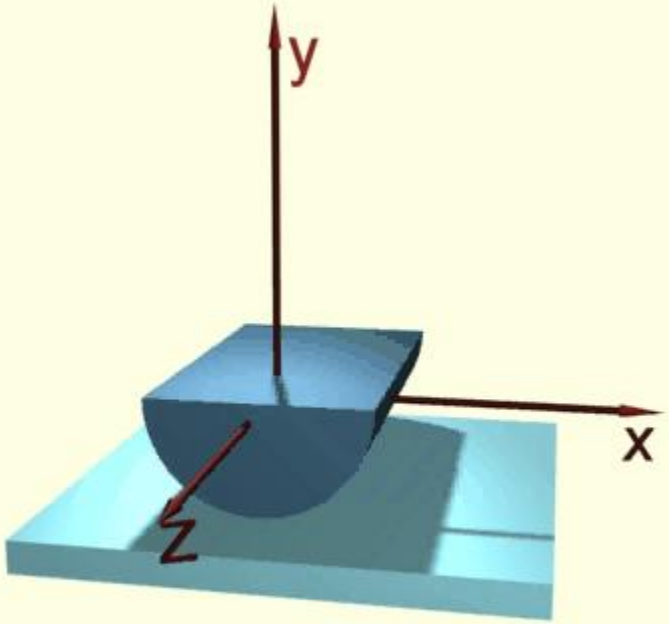
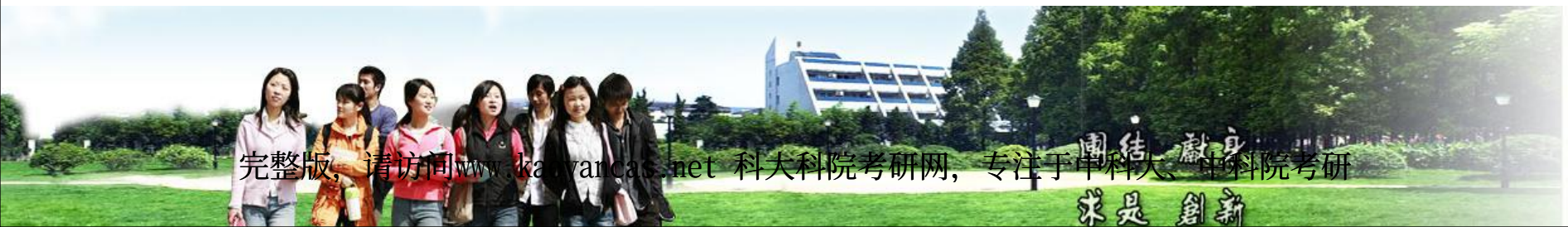
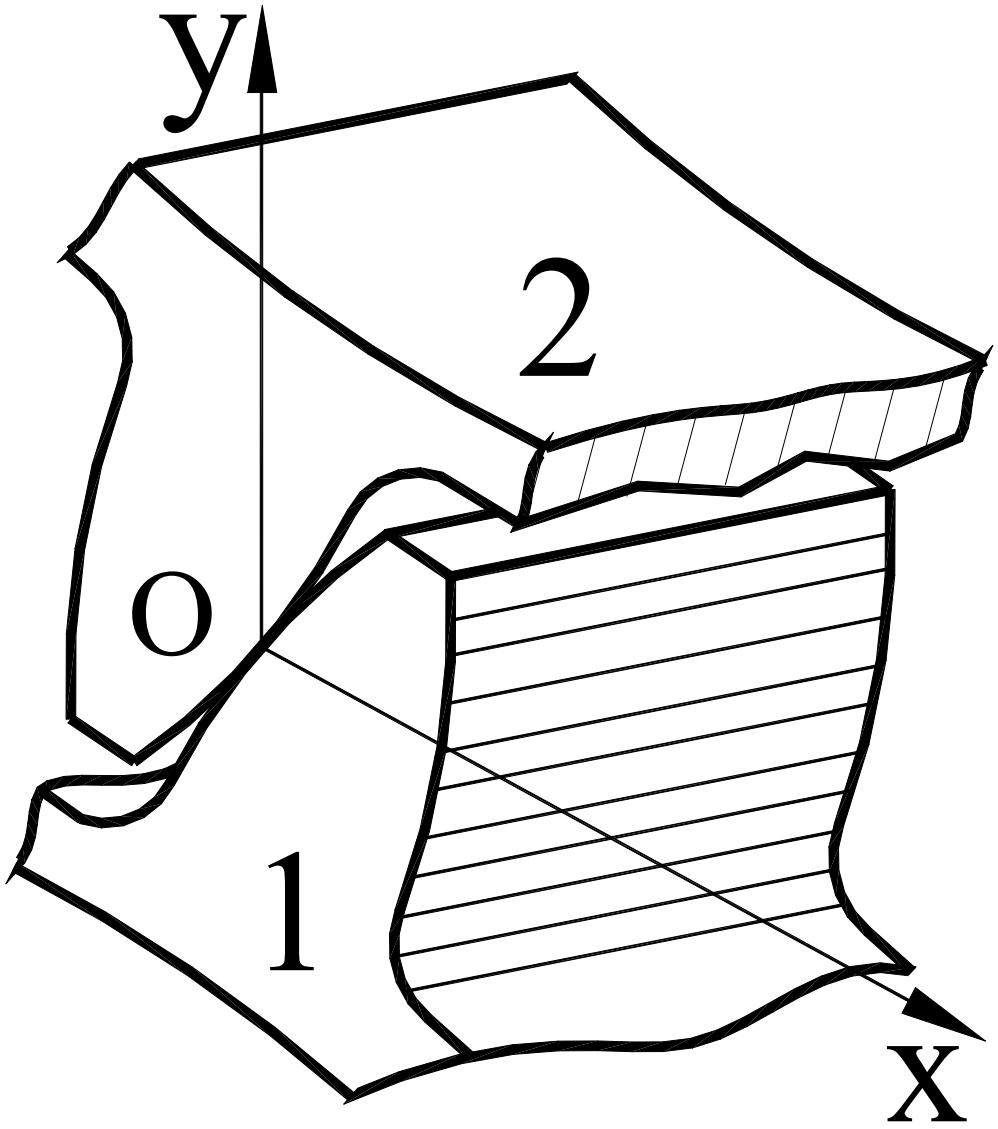
点、线接触——高副higher pair



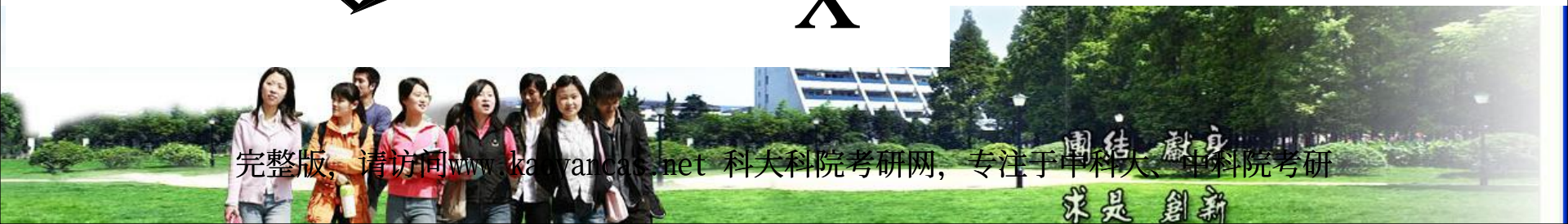


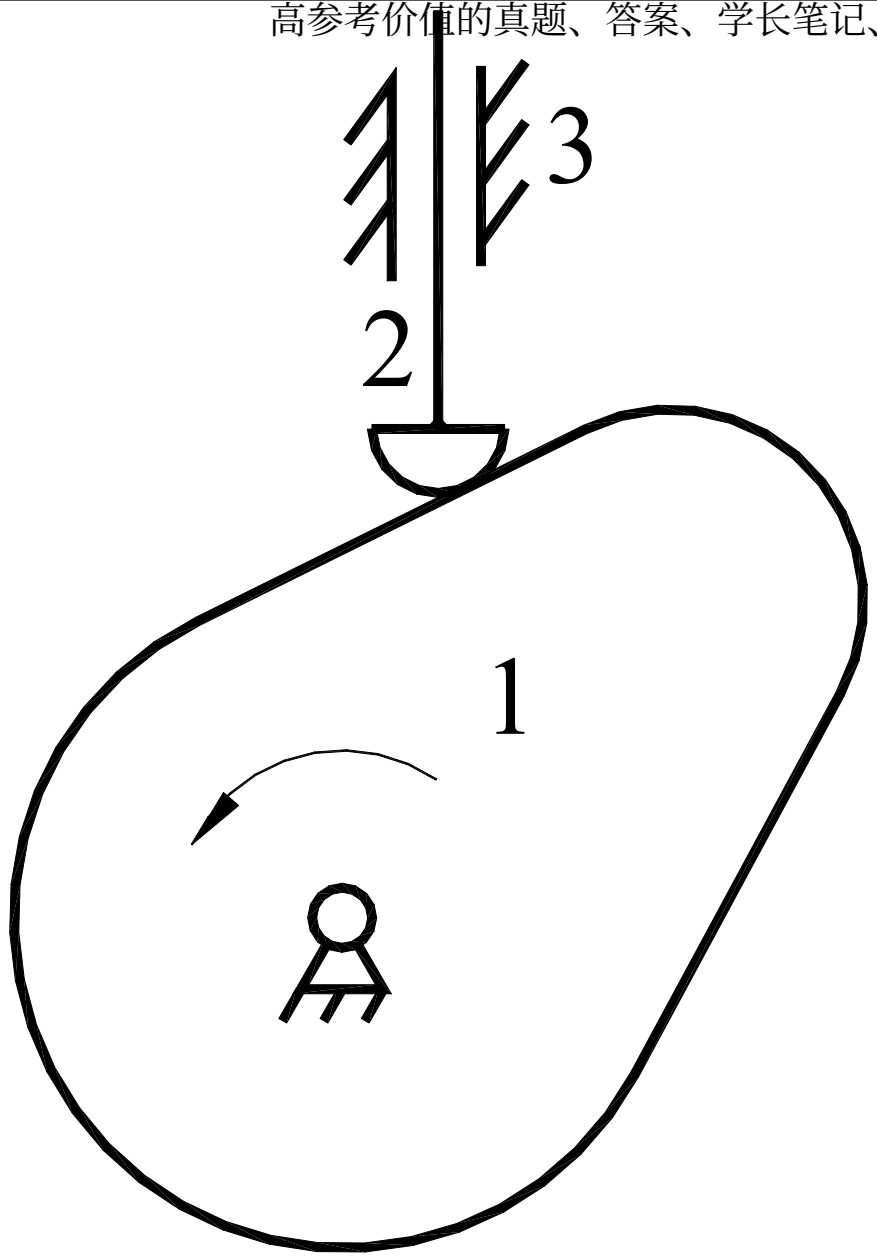




Gear pair

（齿轮副）





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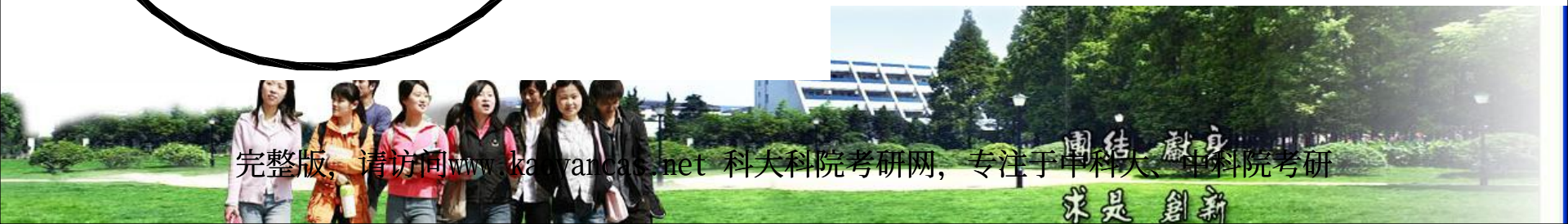
Cam pair
（凸轮副）



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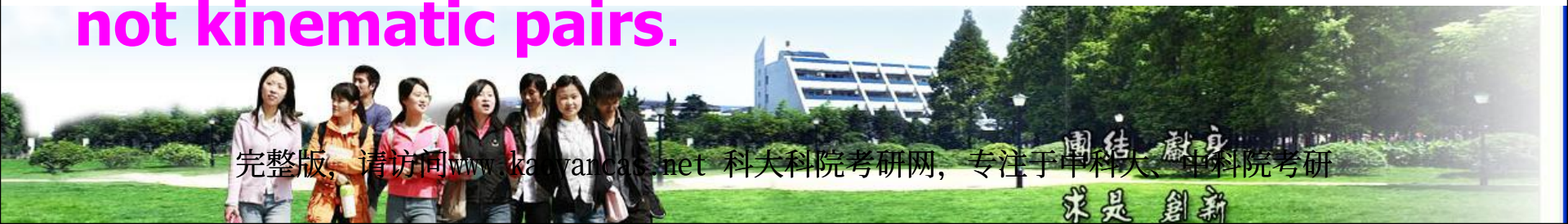
A pair element运动副元素——
the part of the link surface Which make contact with another link

Attention:

（1）One kinematic pair can connect only two links.

（2） Those connections that join two machine elements firmly(牢固地) and do not allow the connected machine elements(零件) to move relative to each other, such as welds(焊接), rivets(铆钉), or nuts(螺母) and bolts(螺栓), are not kinematic pairs.



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Conclusion：Types of kinematic pairs

按两构件间相对运动 {
- planar kinematic pair(平面)
- Spatial kinematic pair(空间)

按两构件接触方式 {
- 面接触(surface contact)：lower pair
  sliding revolute screw spherical
  如：移动副、转动副；螺旋副、球副
- 点、线接触(point, line contact)：higher pair
  如：齿轮副，凸轮副
  gear cam

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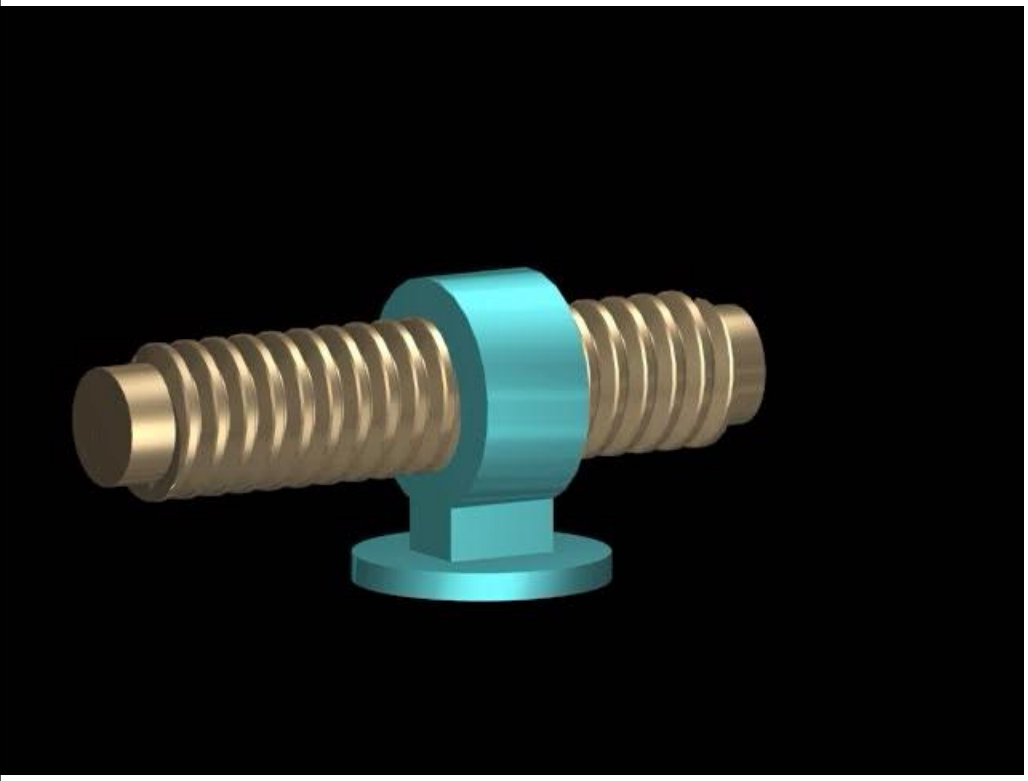
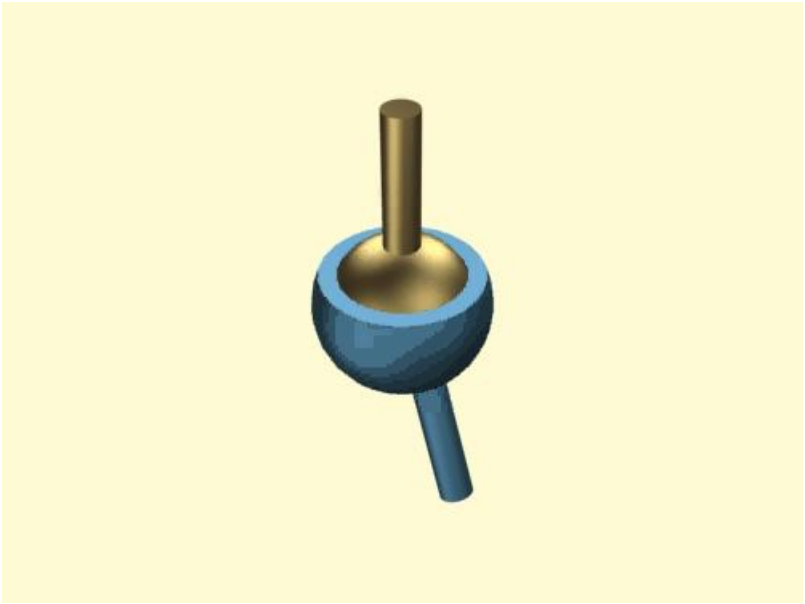
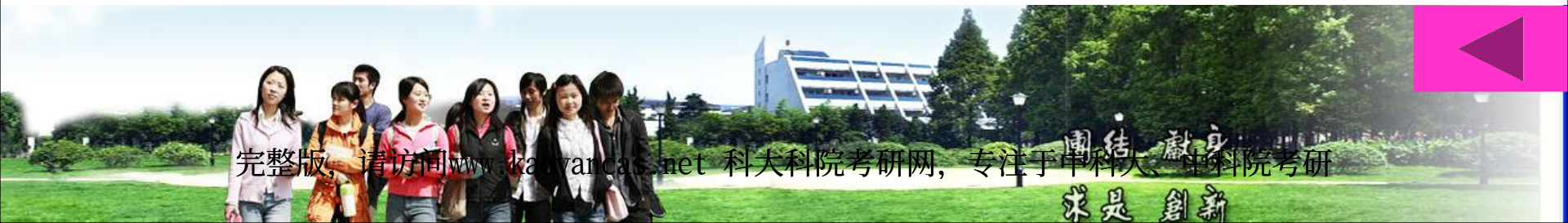




螺旋副
screw

球副
spherical



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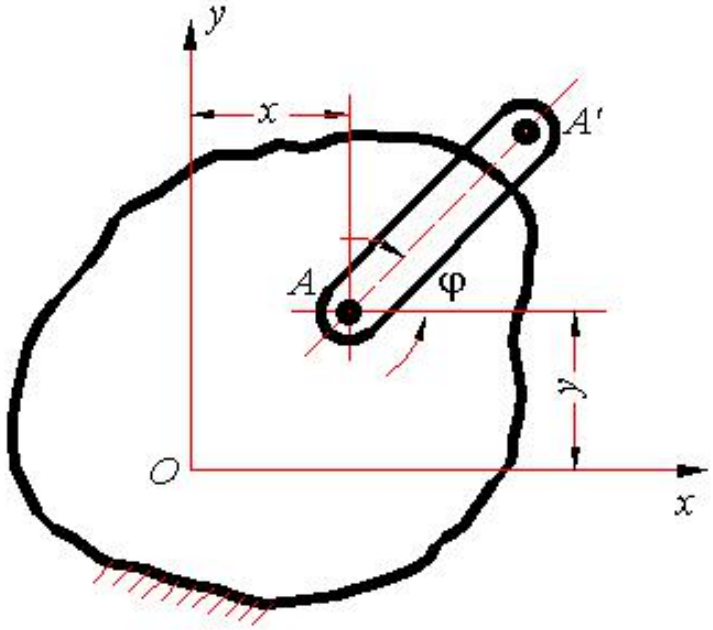
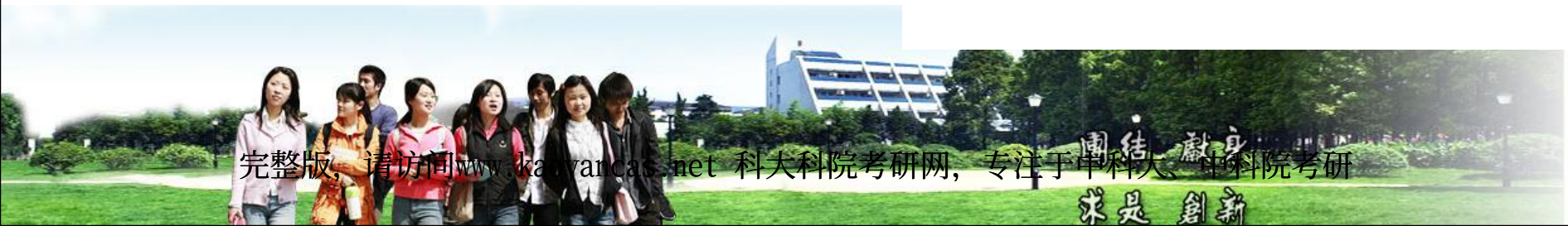
DOF of a Link构件的自由度——

构件含有独立运动的数目

an unconstrained link on a plane would have 3 DOF（刚体作平面运动时，有3个自由度）

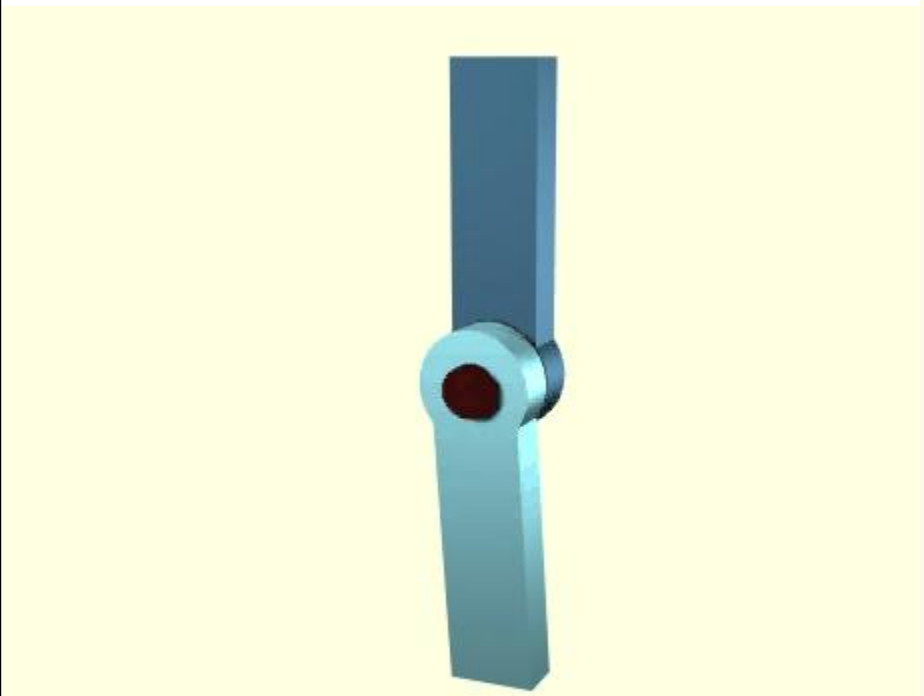
刚体作空间运动时，
则有6个自由度。

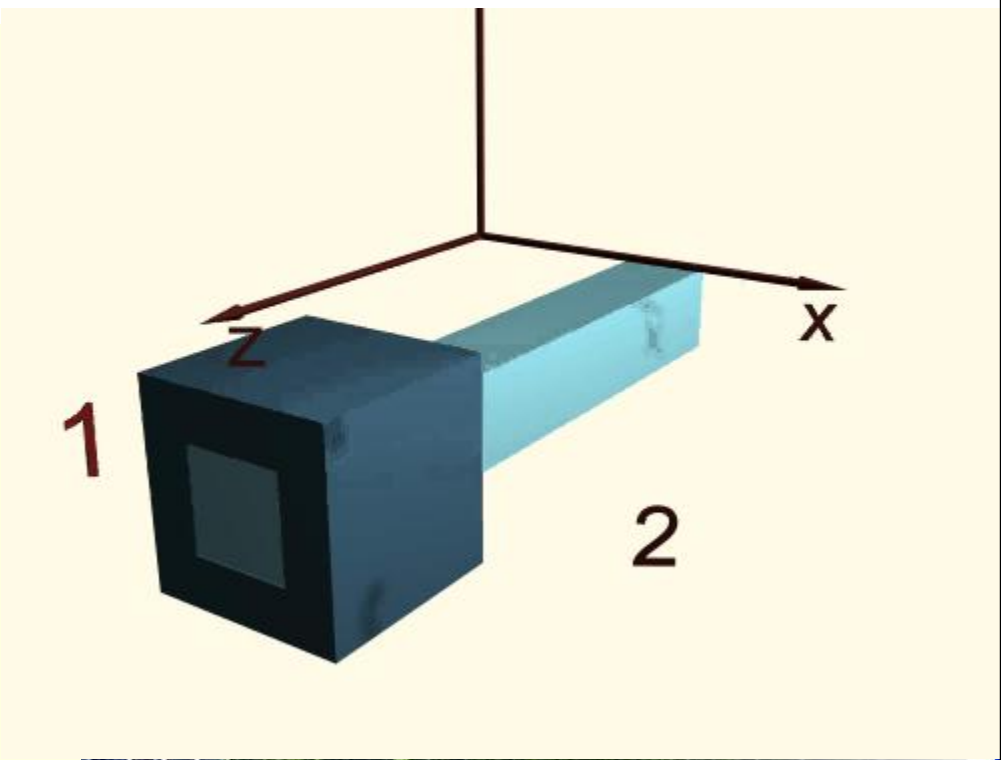




Constraint（约束）：对独立运动的限制

one planar lower pairs（低副）sets two constraints （2个约束，1个自由度）





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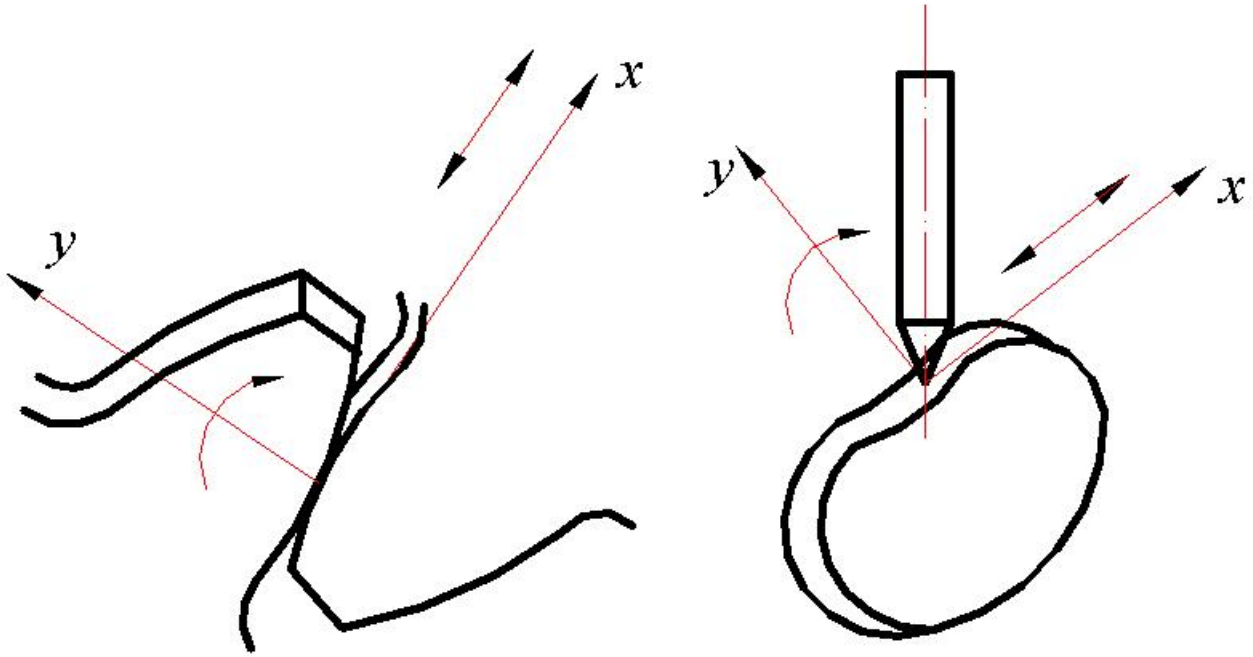
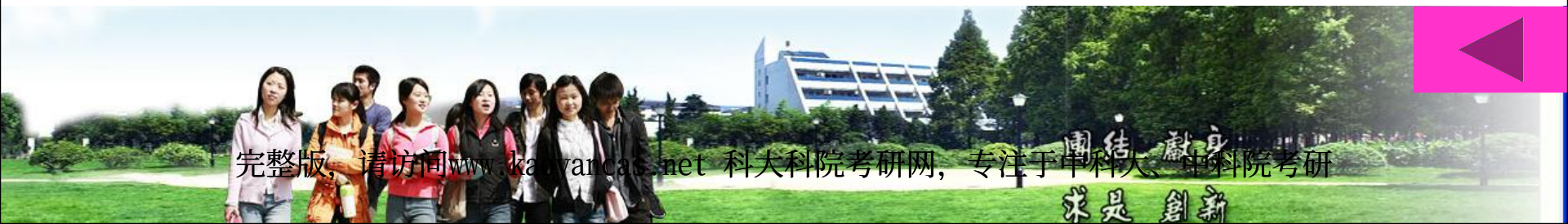
one planar higher pairs（高副）sets one constraints （1个约束，2个自由度）





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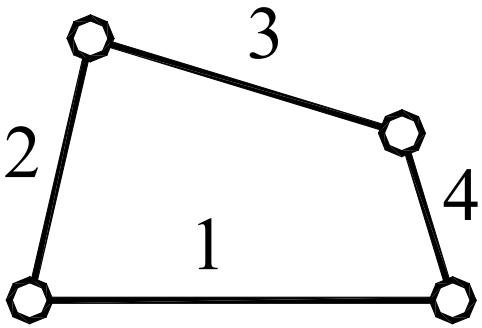
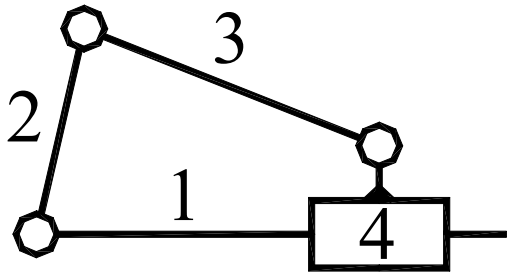
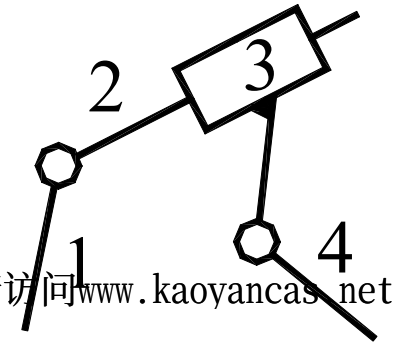
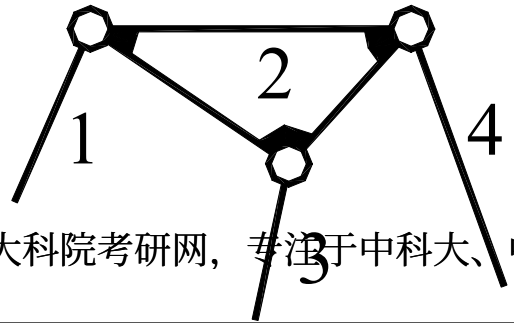
2.2.2. Kinematic Chain(运动链) and Mechanism

When a number of links are connected by means of kinematic pairs, the resulting mobile system is a kinematic chain.









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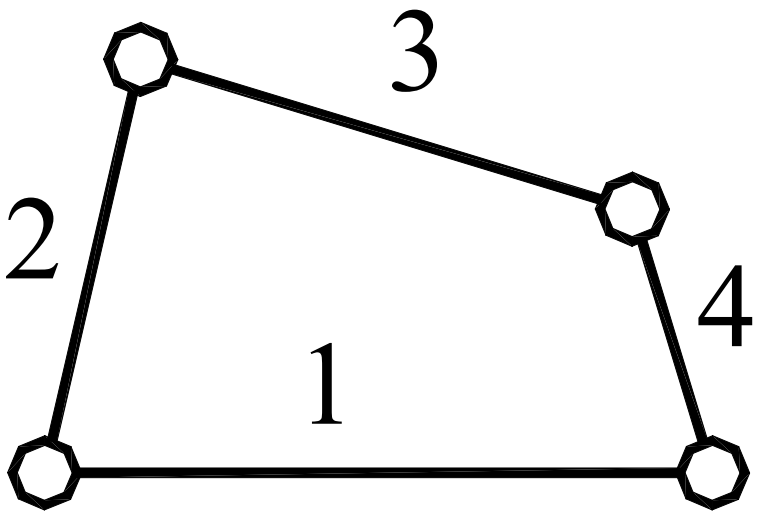
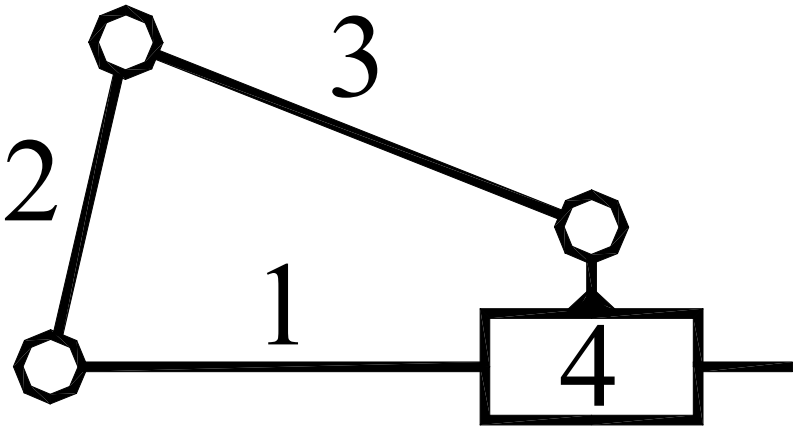
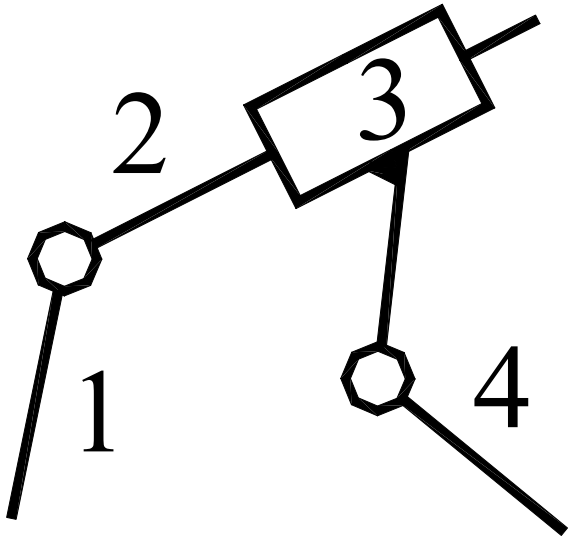
If every link in a kinematic chain has at least two pair elements, all links form a closed chain.

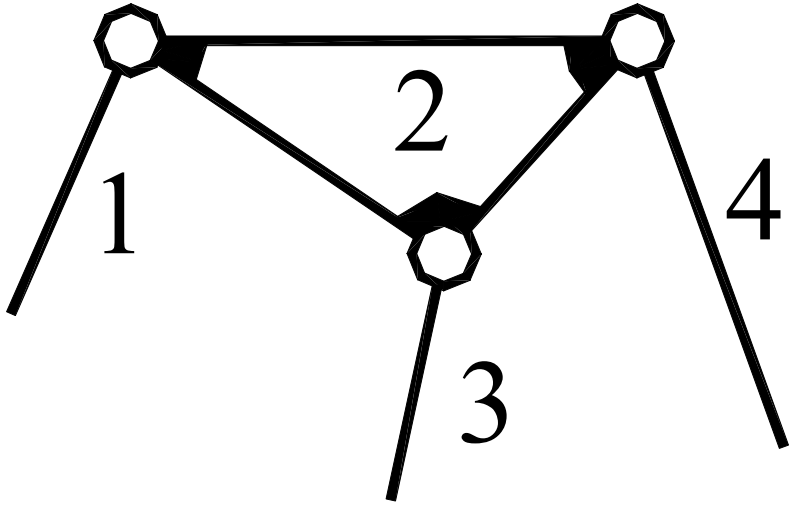
闭链

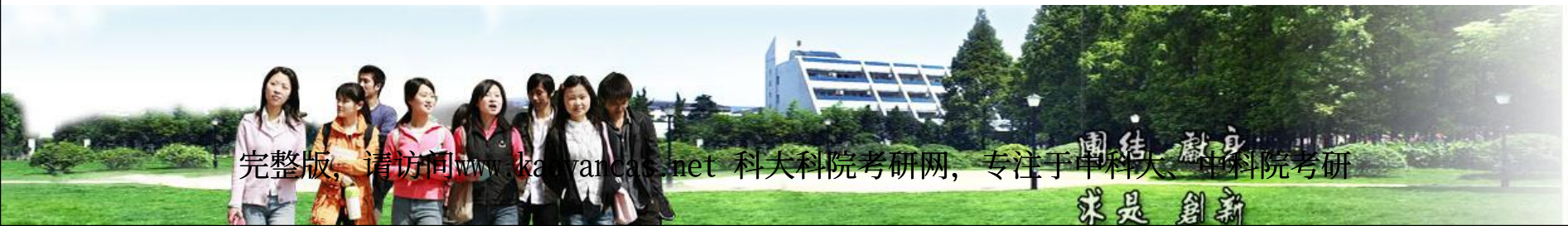




If one or more links in a kinematic chain have only one pair element, then the kinematic chain will be an open chain.开链







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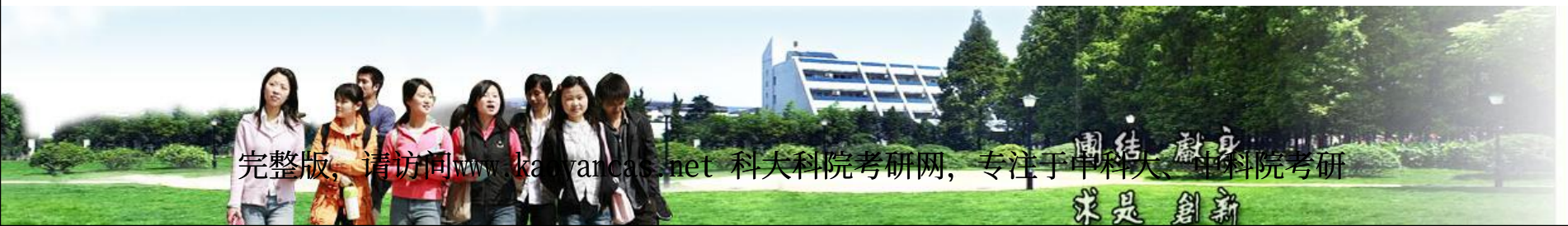
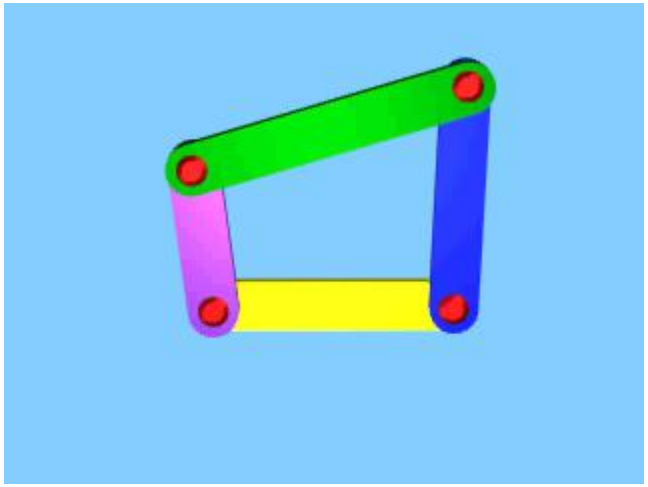
Conclusion：Types of kinematic chain

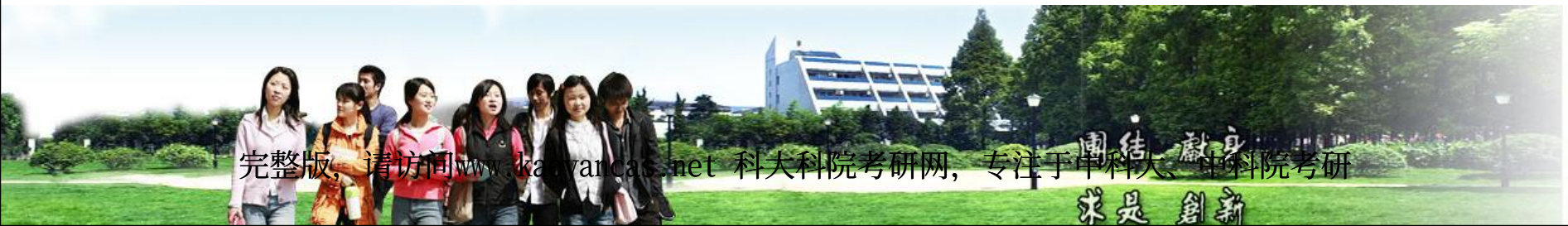
Kinematic chain

- **Closed chain**:（闭链） 每一构件至少含有两个运动副元素,动其中几杆，其余各杆便于传递运动，广泛使用
- **Open chain**:（开链） 一个以上的构件只有一个运动副元素应用于机械手，挖掘机，机器人，多自由度



If a link of the kinematic chain is fixed as the frame机架, then the kinematic chain becomes mechanism.

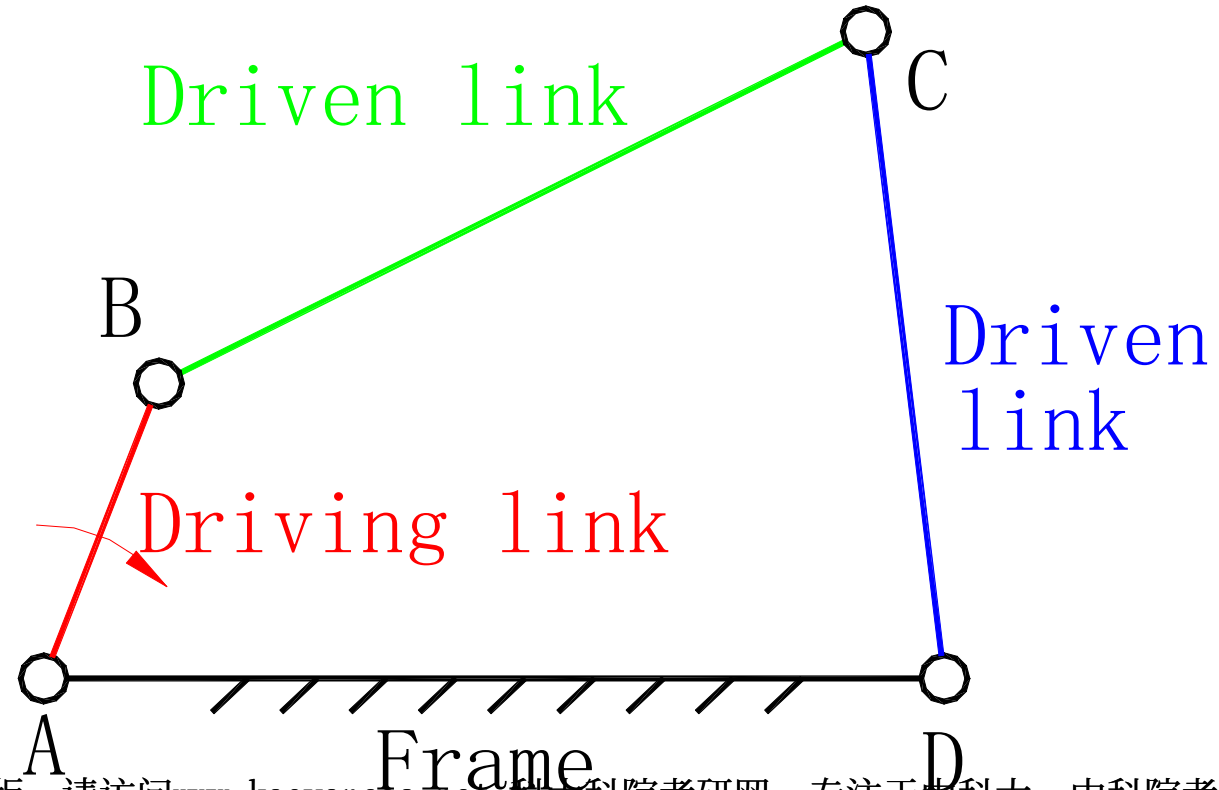




mechanism

driving links:have their own independent motion

driven links:generate the expected output motion ⟹ (output links)



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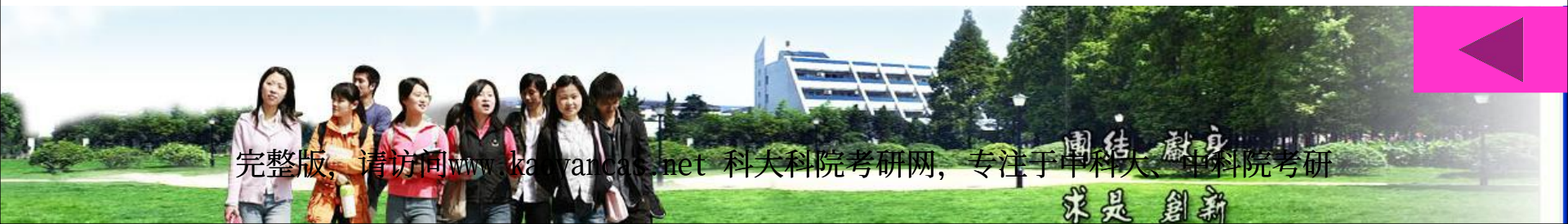
2.3 The Kinematic Diagram of a Mechanism

2.3.1 Definition

2.3.2 Representation of a Kinematic Pair

2.3.3 The representation of a link in the kinematic Diagram

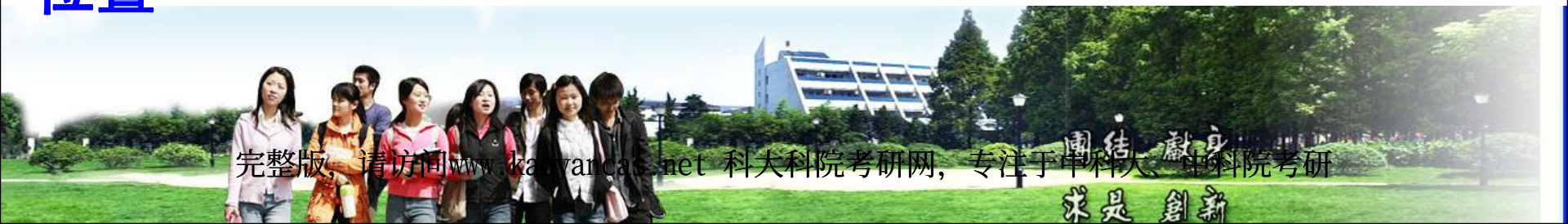
2.3.4 Procedures for Drawing the Kinematic Diagram of a Mechanism



2.3.1 Definition

this diagram is used only to express the relationship between the motions of links, it should be simple but provide all necessary (but not redundant多余的) information determining the relative motion of all links. Such a diagram is called the kinematic diagram of the mechanism(机构运动简图).

不考虑与运动无关的因素（外形，尺寸，具体构造），仅用简单线条，符号，按一定比例表示各运动副的相对位置



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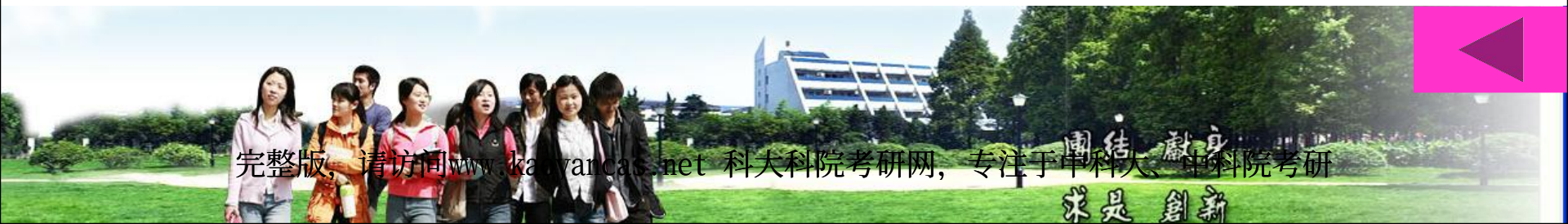
Attention:

- It should be simple but provide all necessary (but not redundant) information determining the relative motion of all links.
- According to specified symbols(符号).
- The kinematic diagram should be drawn to scale(按比例). If not, such a diagram is called the schematic diagram(机构示意图) of the mechanism.



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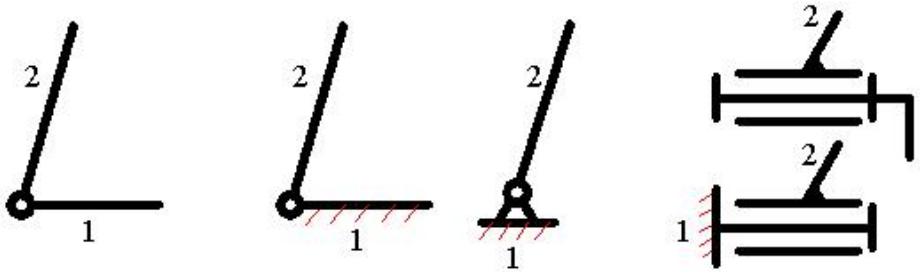
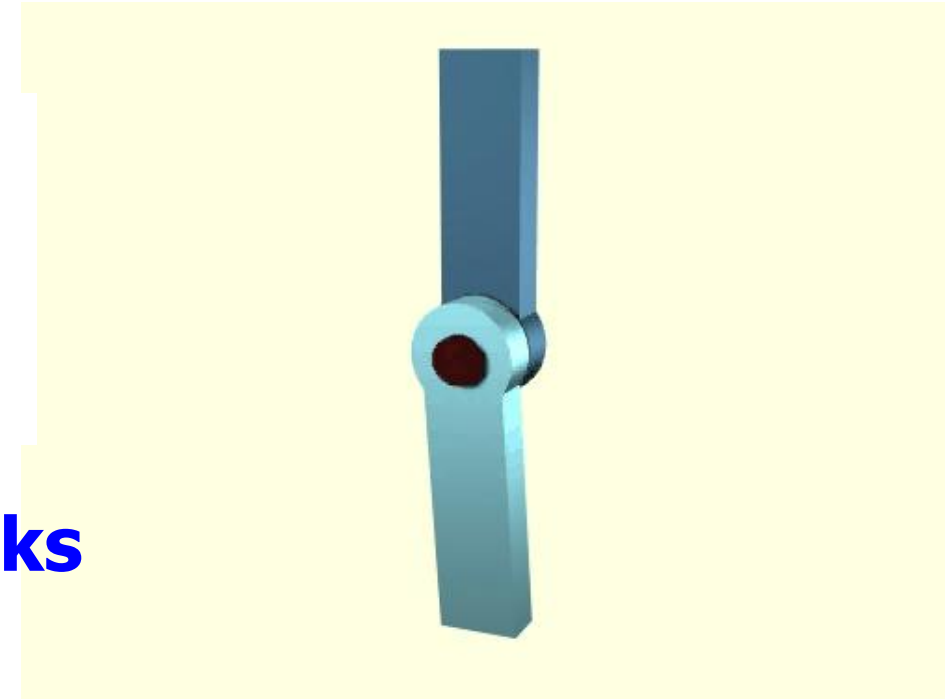
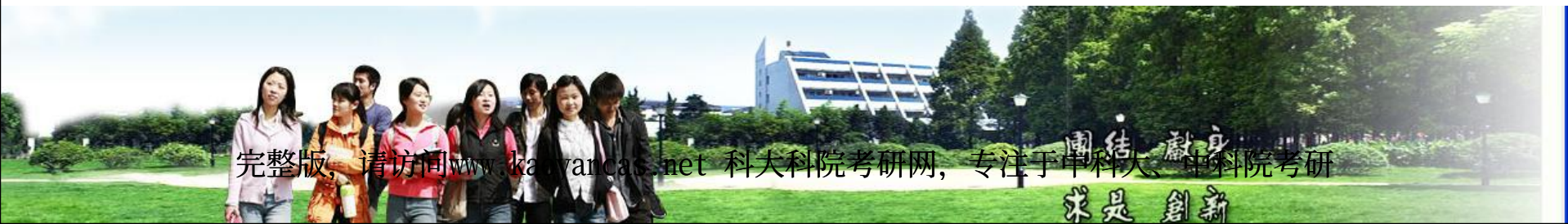
2.3.2 Representation of a Kinematic Pair

转动副 revolute pair：(关键画出转轴位置)

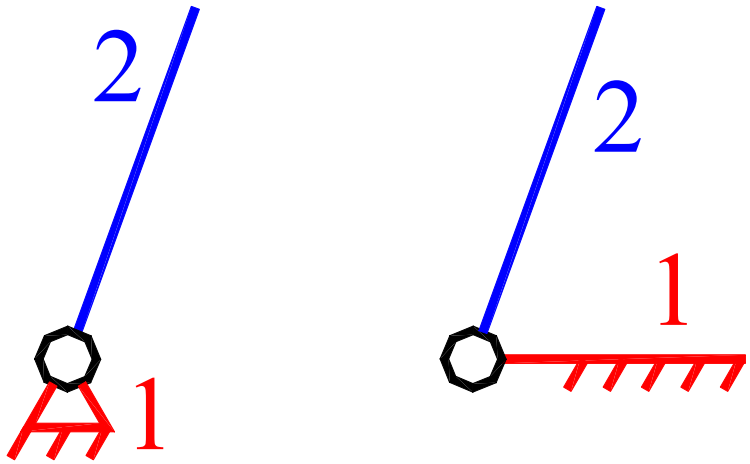


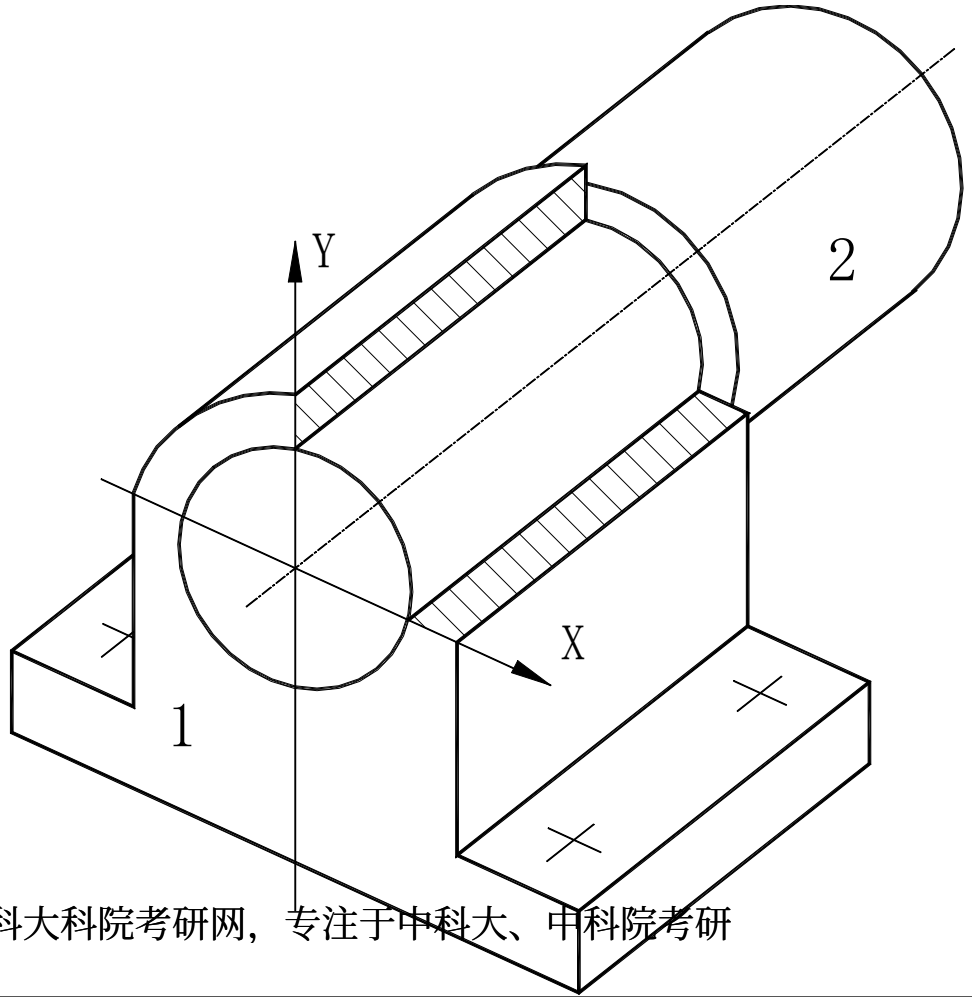


The shaded(带阴影线) links represent the frame



A revolute is conveniently(方便地) represented by a small circle placed at the centre of the revolute no matter how large its radius(半径) is.





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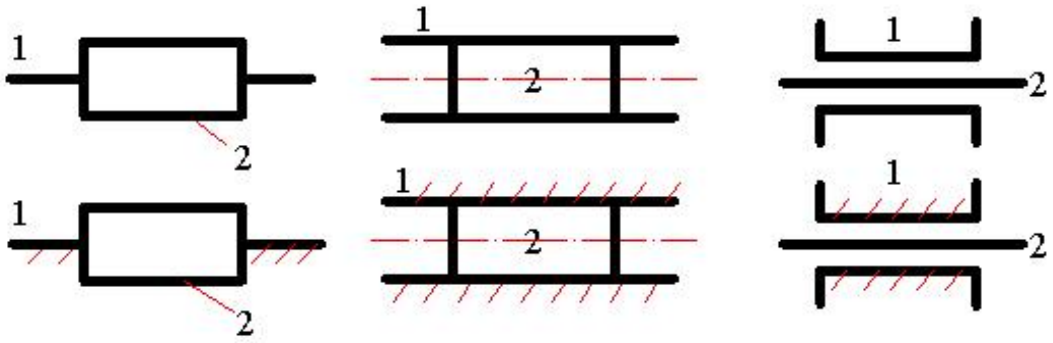
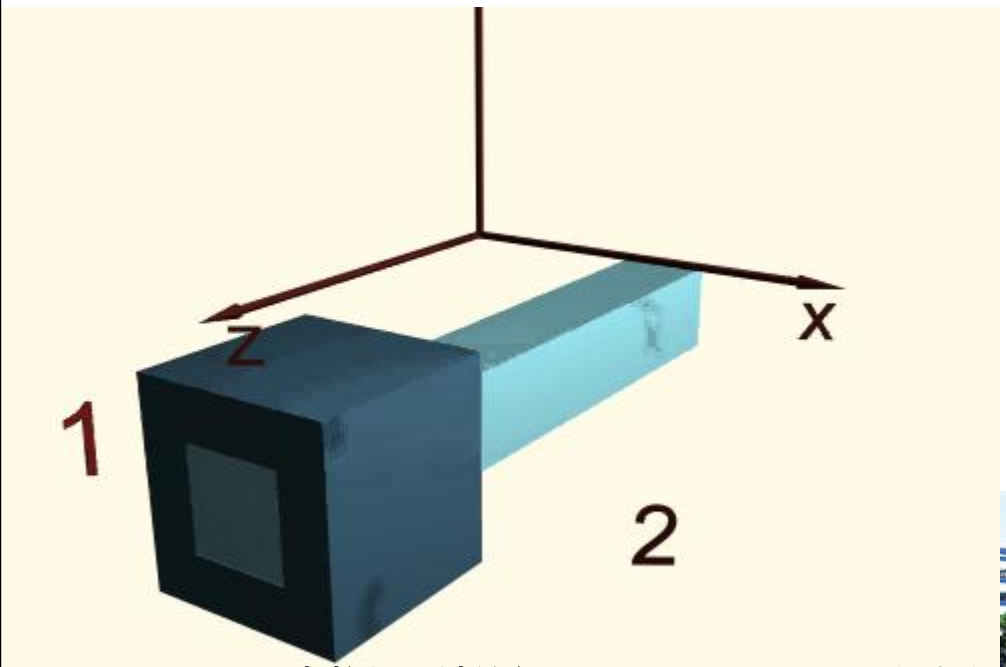
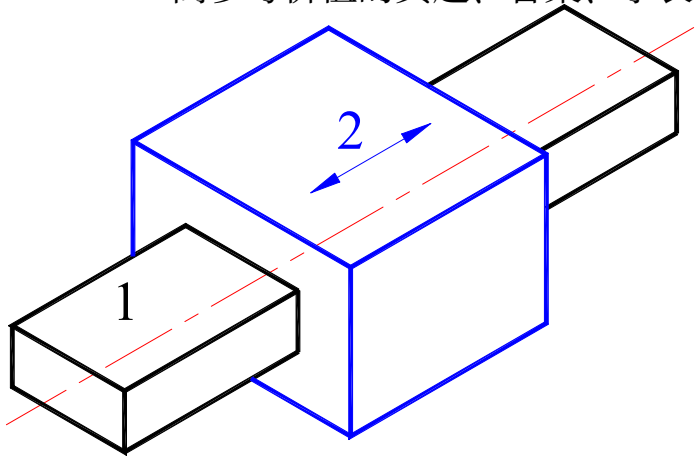
移动副 sliding pair (关键画出相对移动的方向，导路)

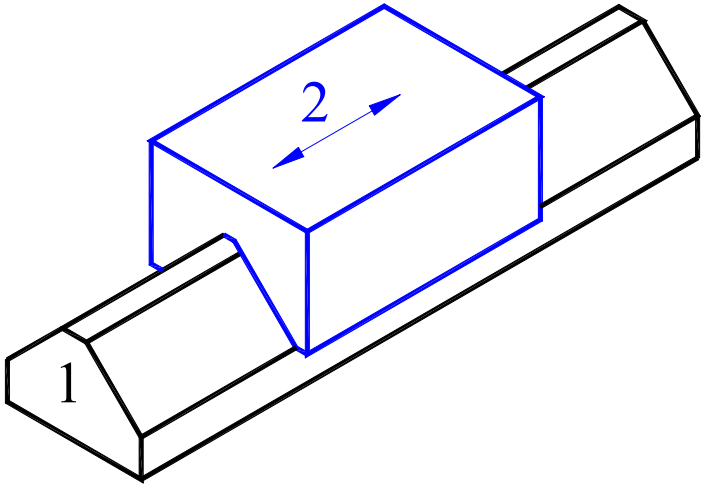




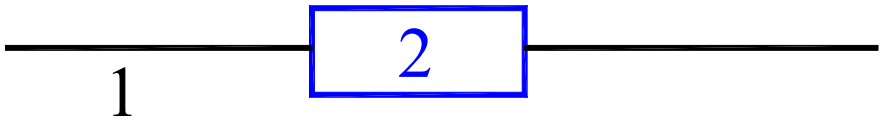
The shaded(带阴影线) links represent the frame



(a)

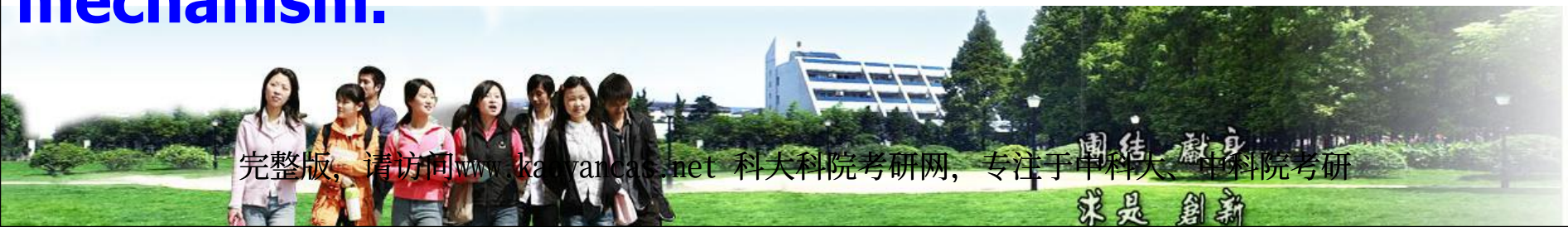


(b)



(c)

The actual shape and dimensions of the cross section(横截面) of the sliding pair has no influence(影响) on the kinematics of the mechanism.



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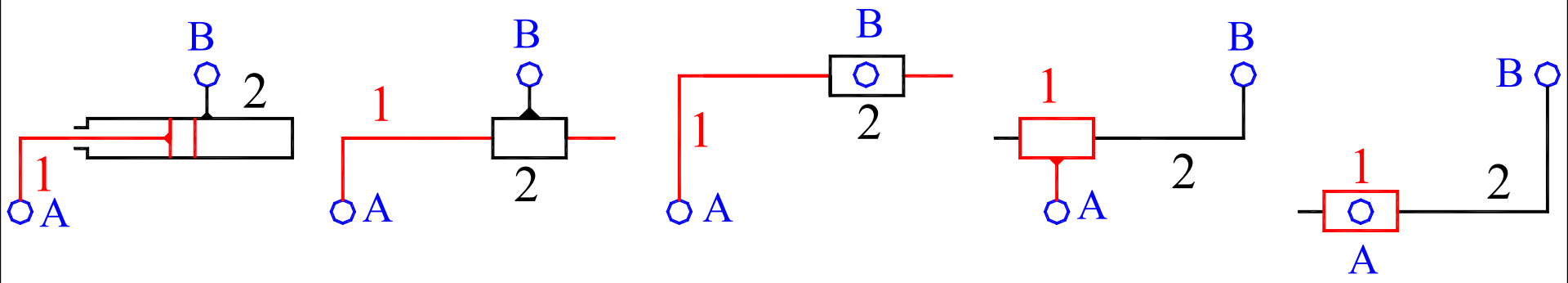
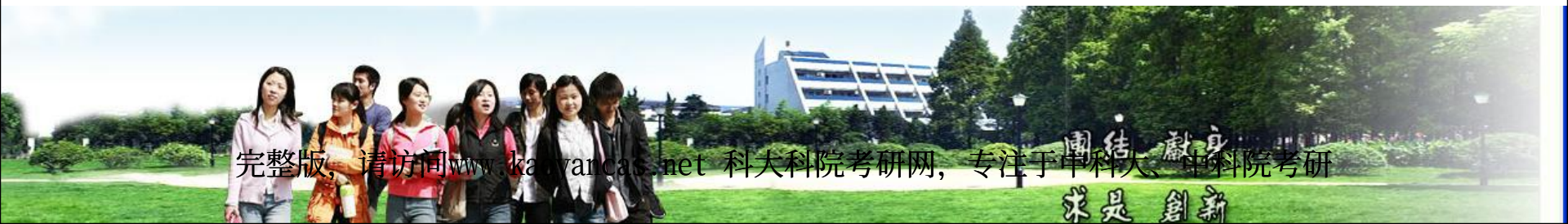
Attention：

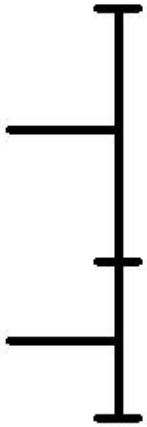
- The centerline of the sliding pair in the kinematic diagram must be parallel(平行) to the pathway(导路) in the mechanism.
- Any one link can be drawn as a sliding block while the other is drawn as a guide bar(导杆).

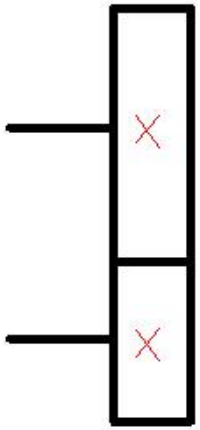
The following five diagrams are equivalent.

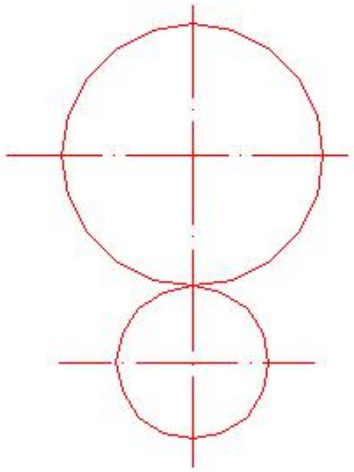




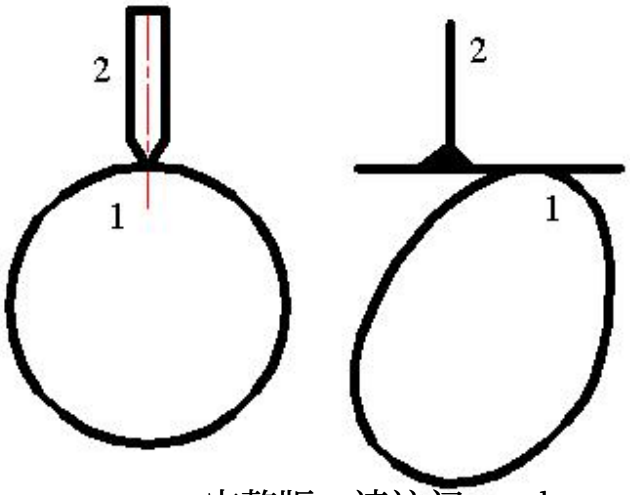
齿轮副gear pair：







凸轮副cam：

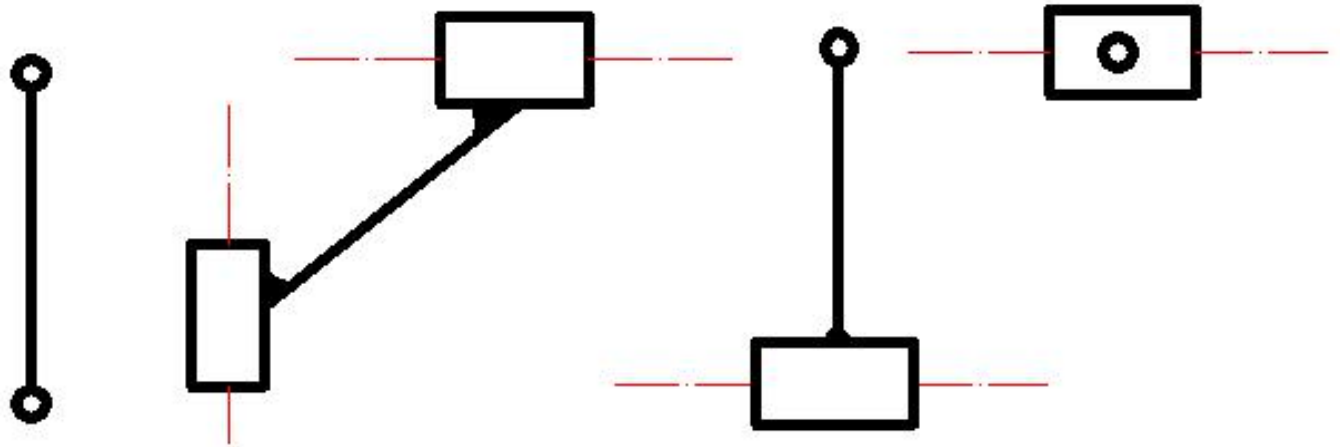


need actual cam contour and end profile of follower

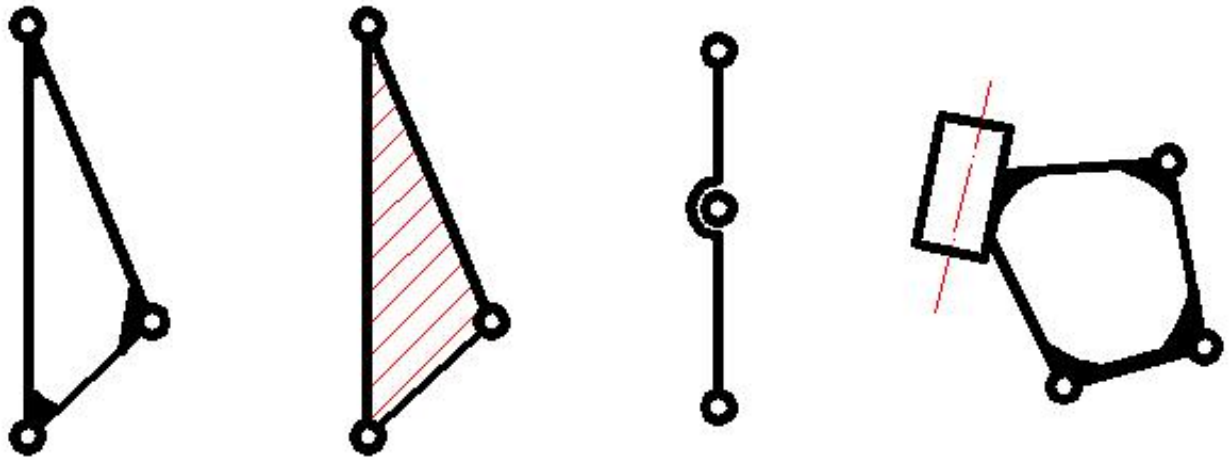
2.3.3 The representation of a link in the kinematic Diagram

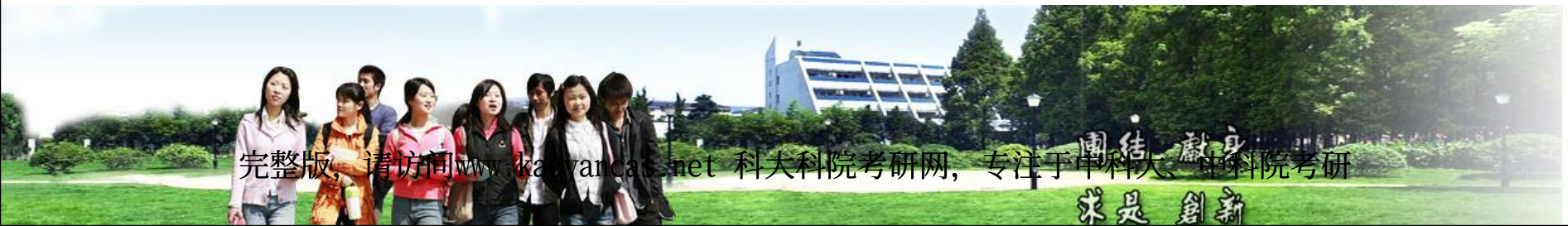
the kinematic function of the link to keep the distance ∴straight line can be used

links with two pair elements(双副构件)：



A link with more than two pair elements can be represented by a hatched(阴影线的) or welded(焊接的) polygon(多边形) with pair elements at corners.





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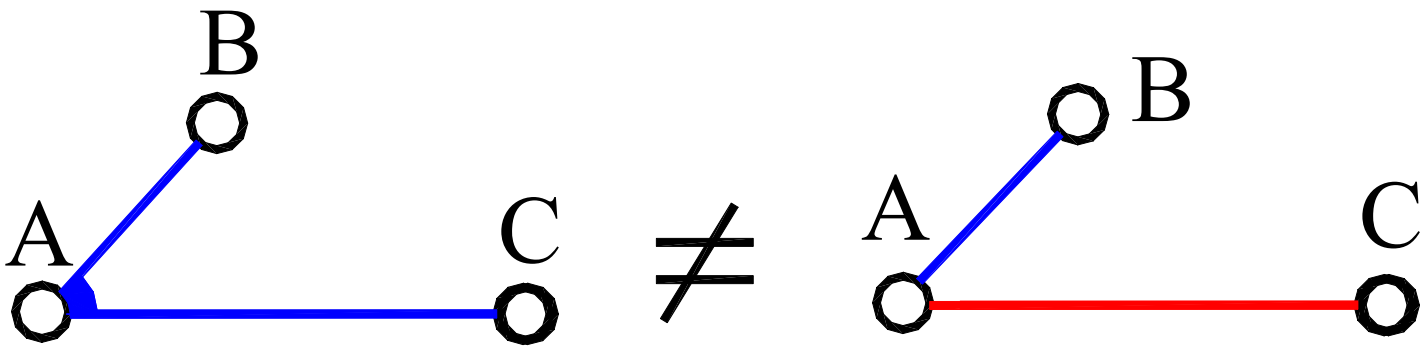
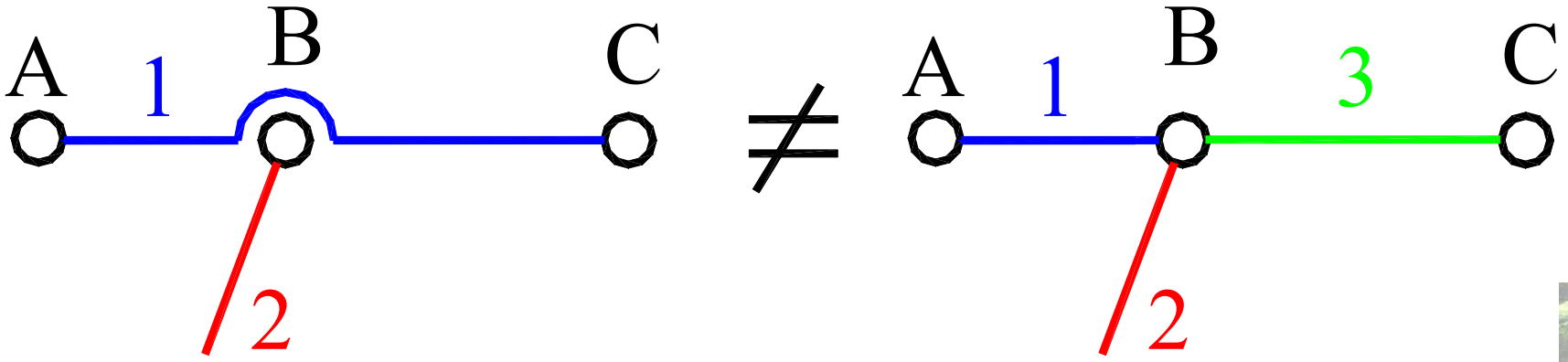
Note the differences between the following two groups of figures：





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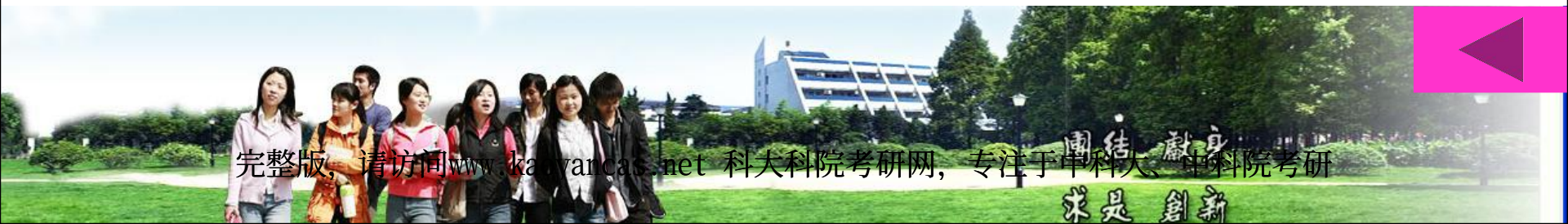
The special representative(有代表性的) symbols(符号) used in a kinematic diagram for some common mechanisms are listed in Table 2-1（P9）.



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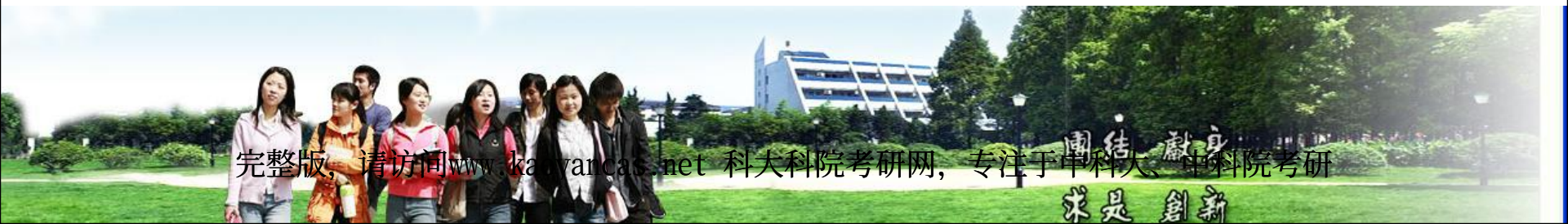
2.3.4 Procedures for Drawing the Kinematic Diagram of a Mechanism

1) Run the mechanism slowly,study the structure of the mechanism, How many links? Types of all kinematic pairs?

2) stop at a suitable position ,Chose a drawing plane

3) Draw the schematic diagram机构示意图

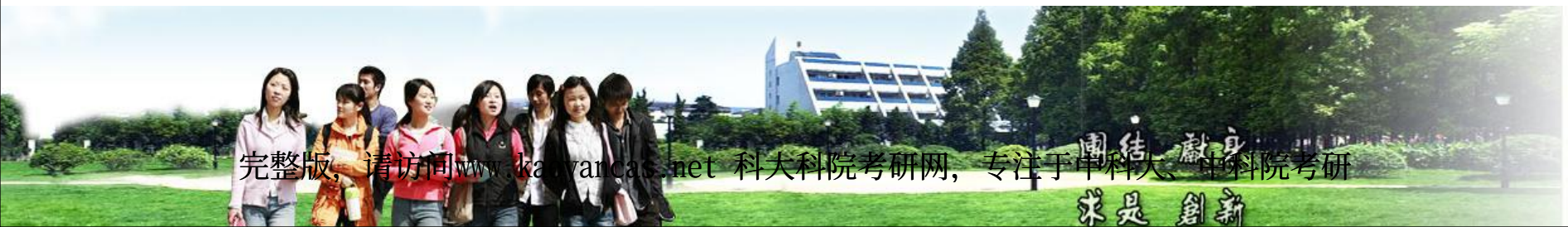
4) Measure kinematic dimensions



5) Select scale

$$\mu_l = \frac{actual\ \ length(m)}{length\ \ in\ \ diagram(mm)}$$

6)draw the kinematic diagram of the mechanism.



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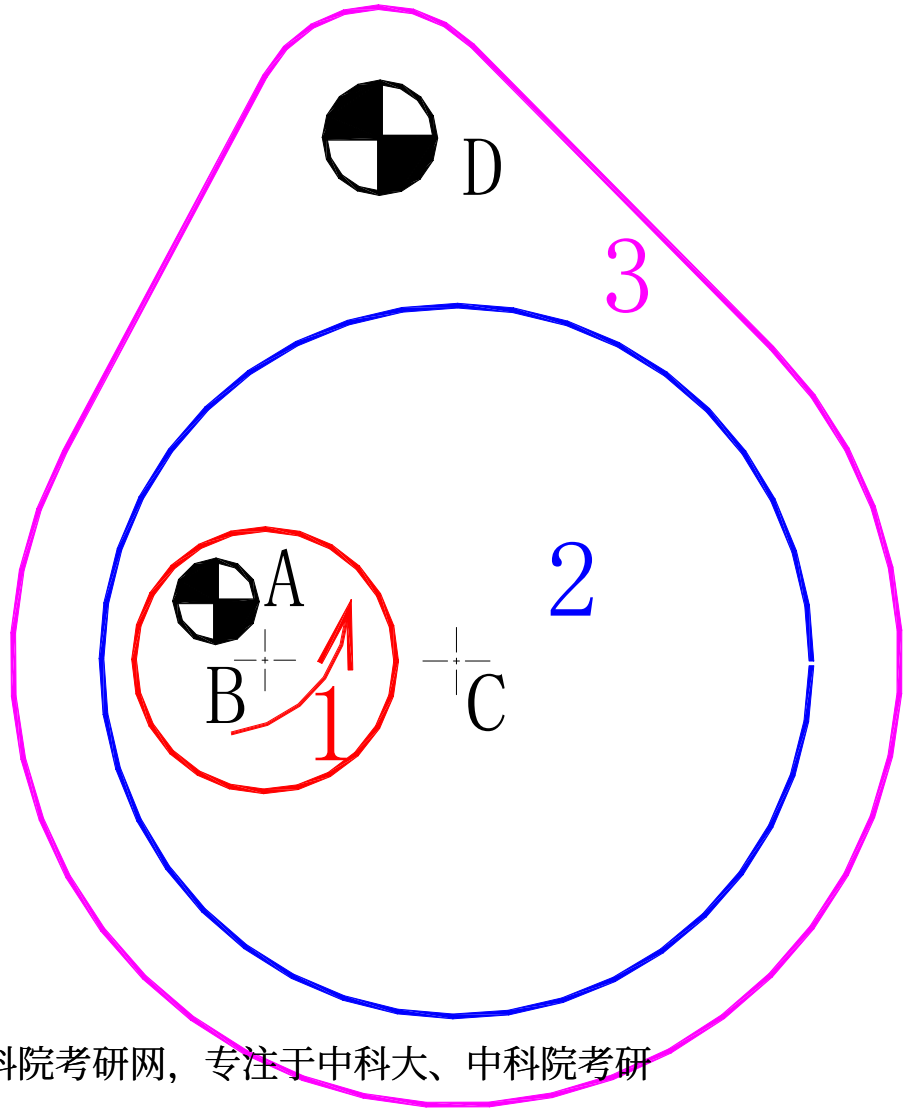
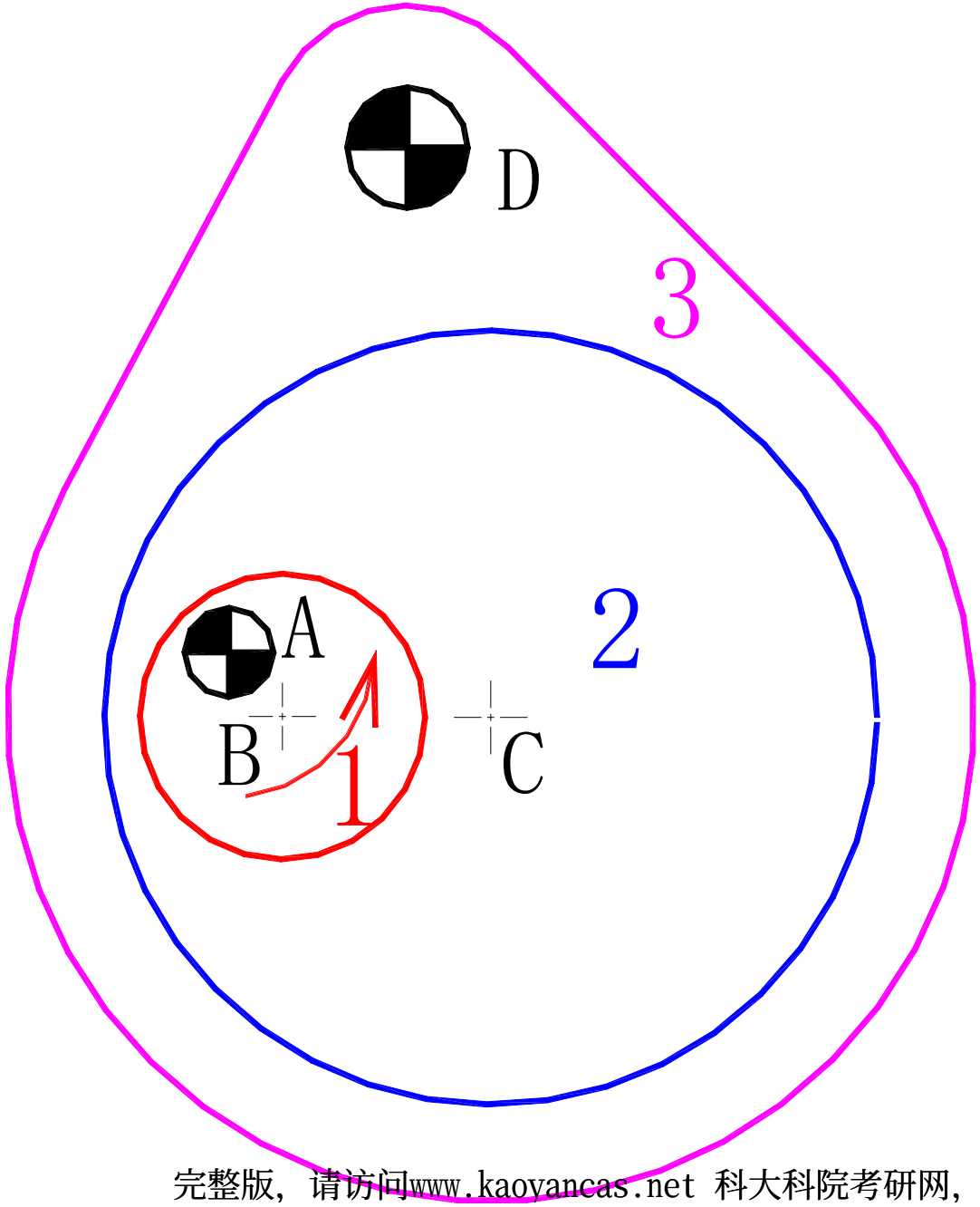
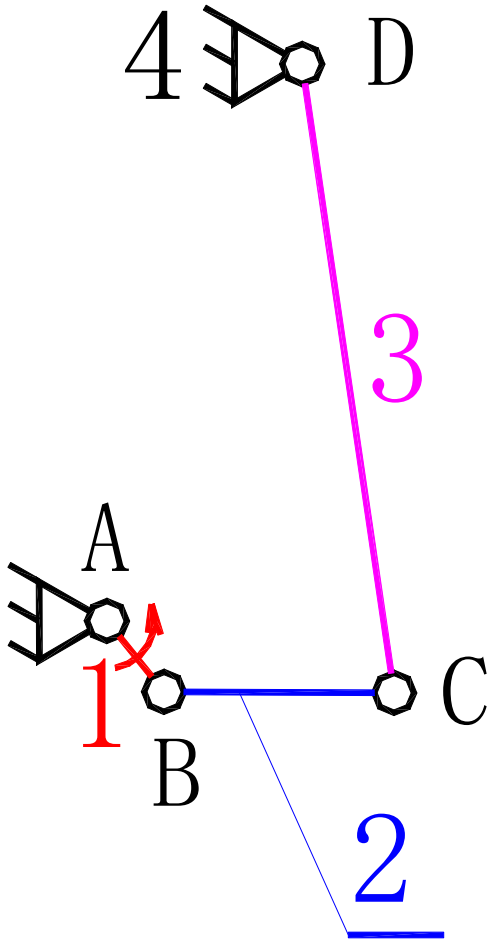
Ex.： Circular disk 1 rotates relative to the frame 4 about a fixed axis(轴) A. The link 3 oscillates(摆动) about a fixed axis D. The circular disk 1 matches(配合) with a hole on circular disk 2. The circular disk 2 matches with a large hole on the link 3.







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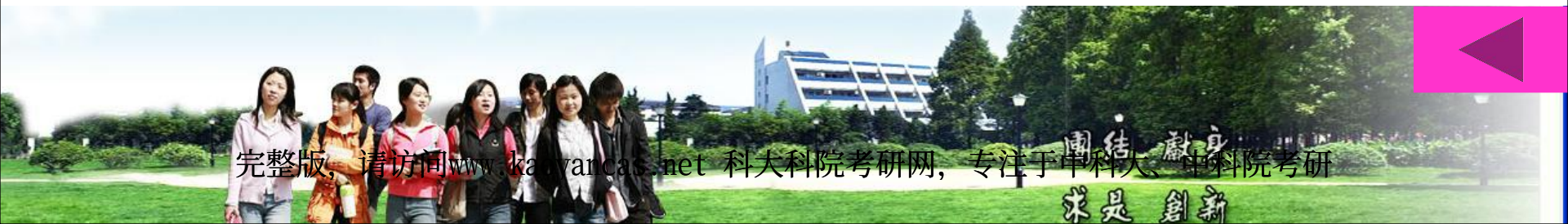
2.4 Degree of Freedom of a Mechanism

2.4.1 the definition of DOF

2.4.2 Structural Formula of a planar Mechanism

2.4.3 Condition for a mechanism to have a determined motion



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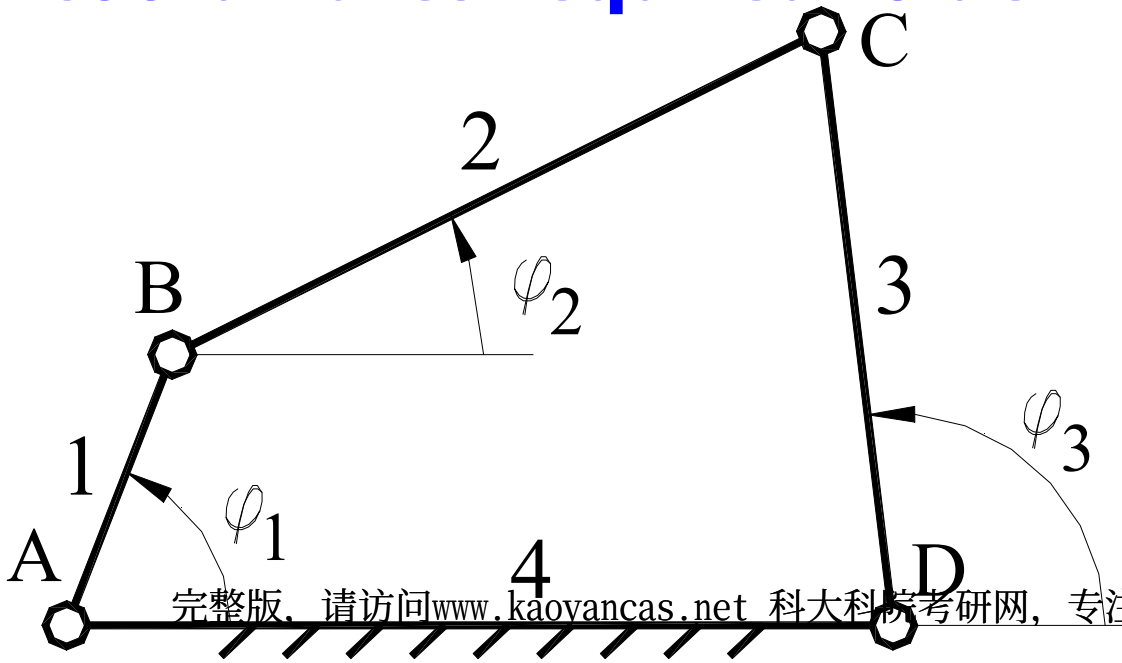
2.4.1 Definition

机构中各构件相对于机架所能有的独立运动的数目

—the DOF of the mechanism

OR

***DOF* : The number of independent coordinates required to define its position.**



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2.4.2 Structural Formula of a Planar Mechanism

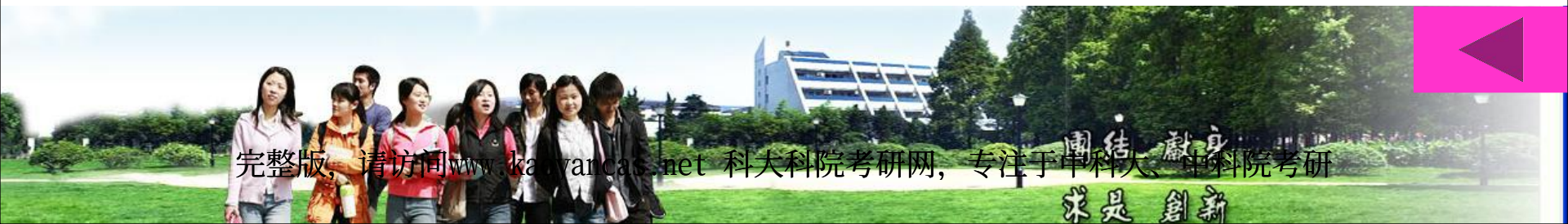
N :the number of moving links (does not include the frame!)

P_L: the number of planar lower pairs

P_h:the number of planar higher pairs

The DOF of a Planar Mechanism

$$F=3N-2P_L-P_h$$



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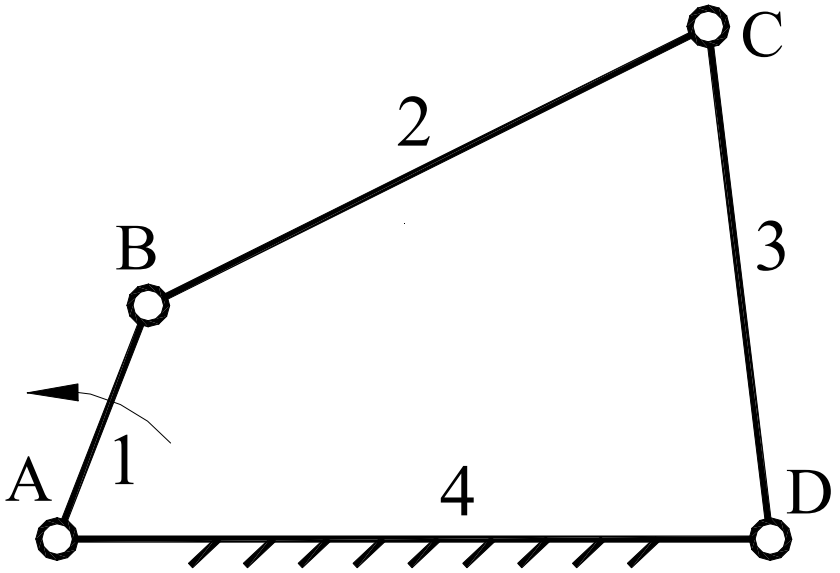
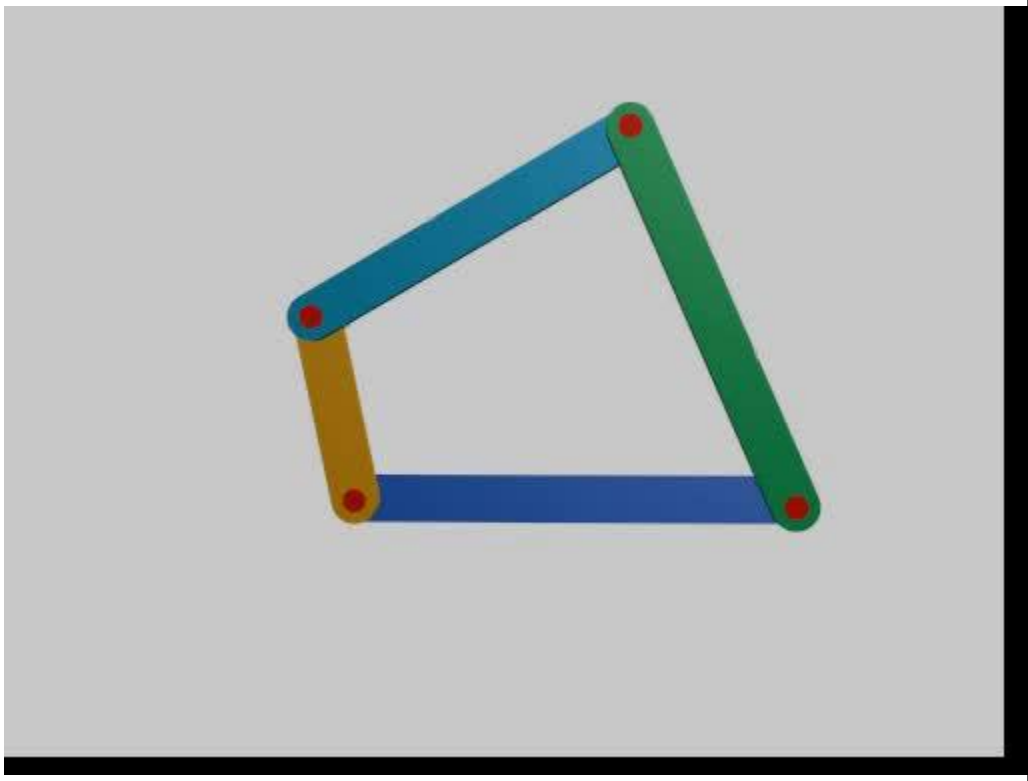
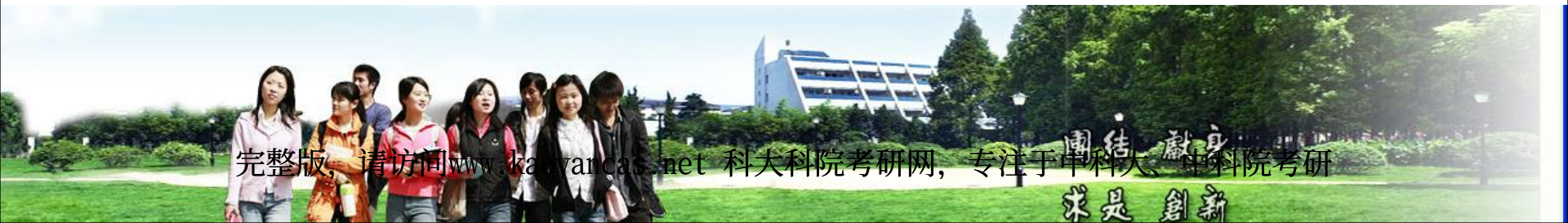
2.4.3 Condition for a mechanism to have a determined motion

Example1:





$$F = 3\times 3 - 2\times 4 - 0 = 1$$



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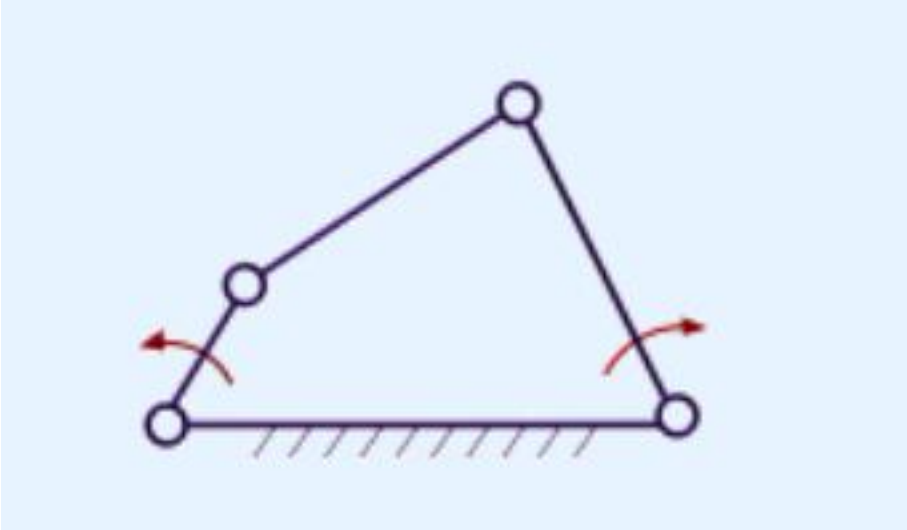
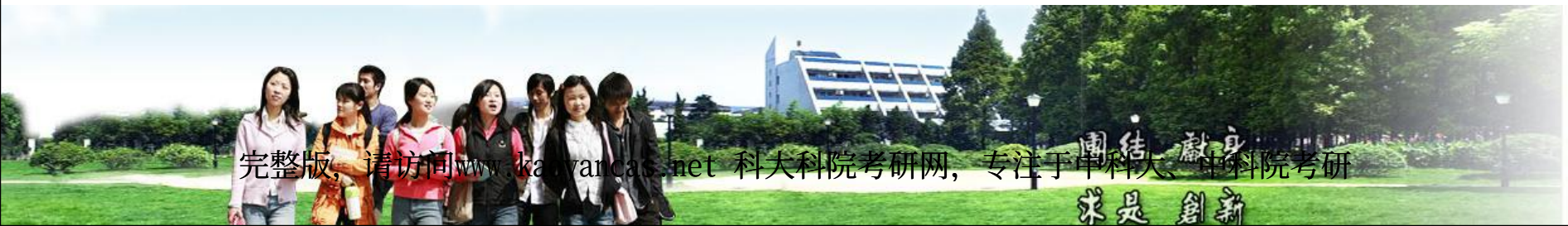
The number of the driving links>the DOF of Mechanism, it will be broken



如果有两个原动件，那么该机构会从最弱构件处损坏。



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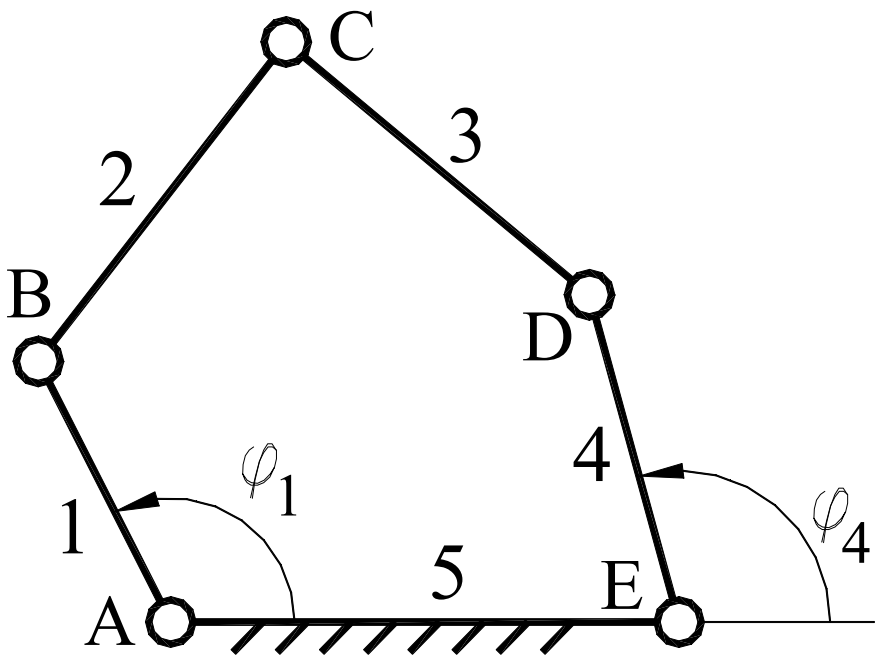
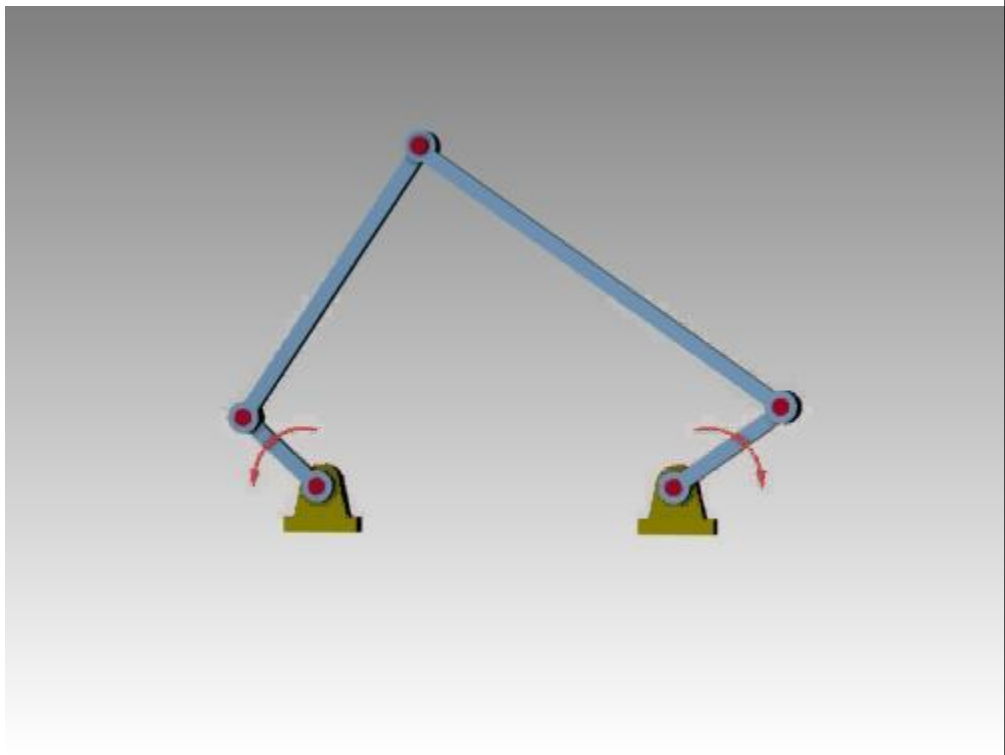
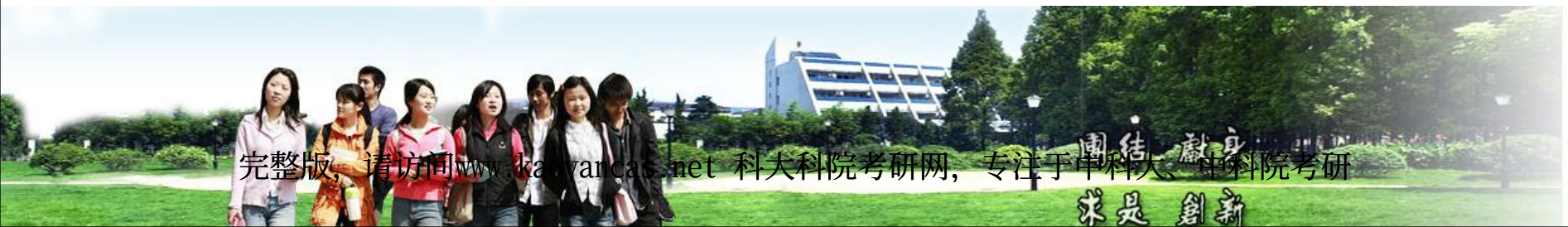
Example2: 铰链五杆机构：





$$F = 3\times 4 - 2\times 5 - 0 = 2$$



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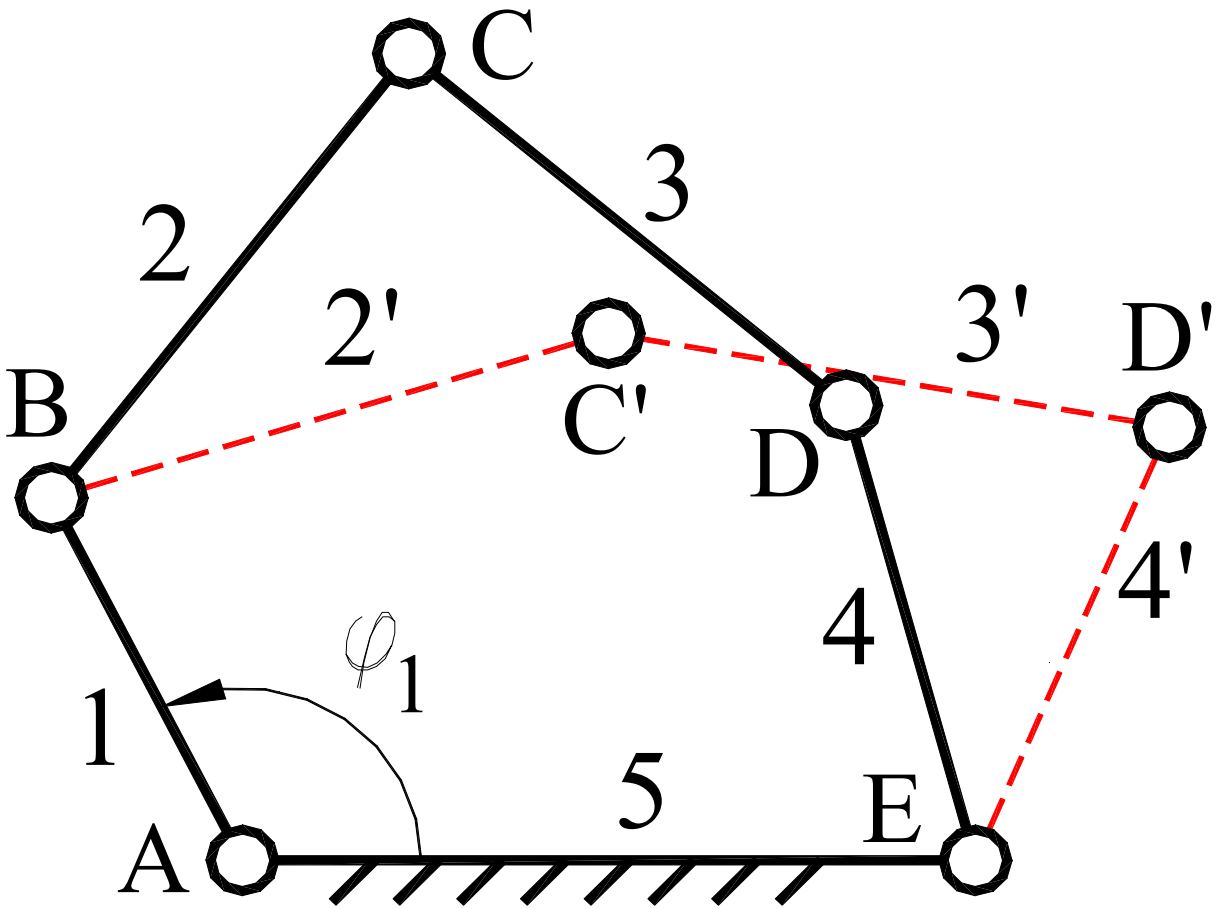
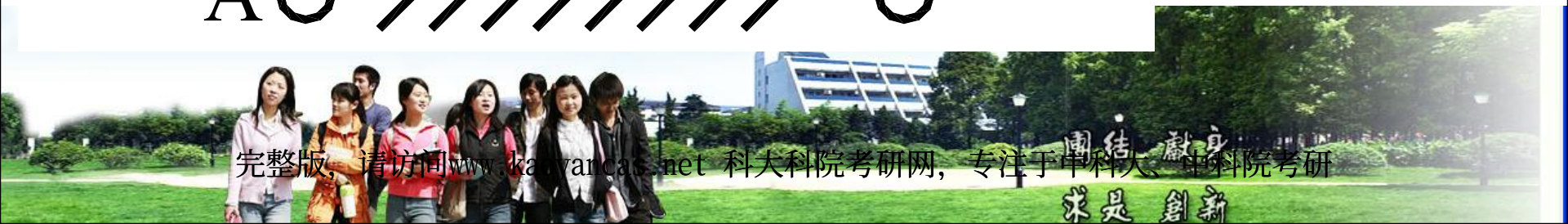
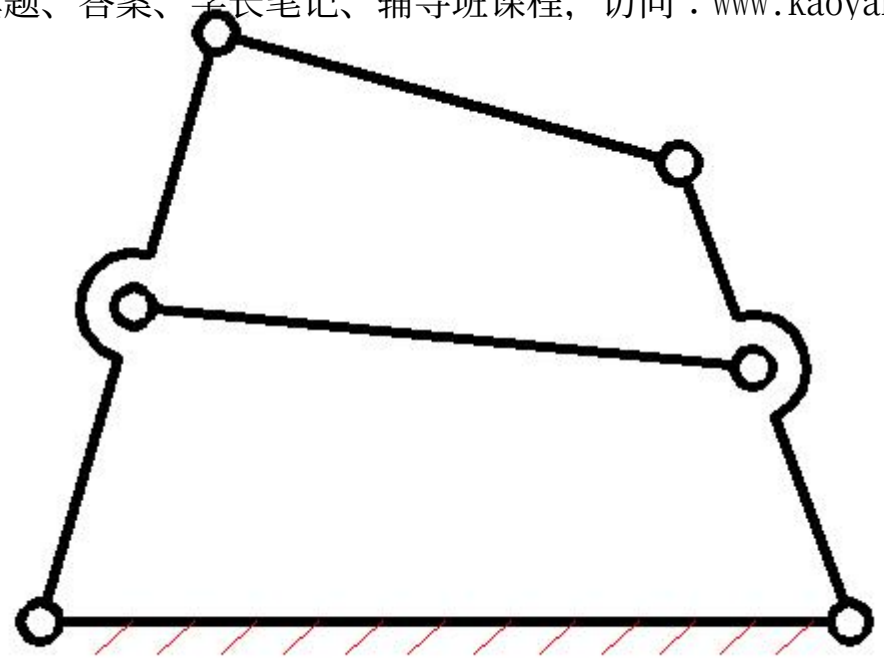
the number of the driving links <the DOF of Mechanism,the motion is not determined



如果只给一个原动件，机构运动不确定



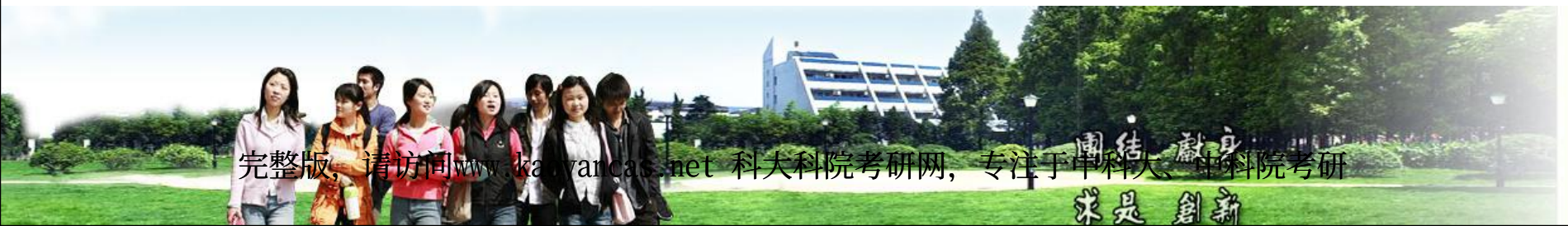
Example3:

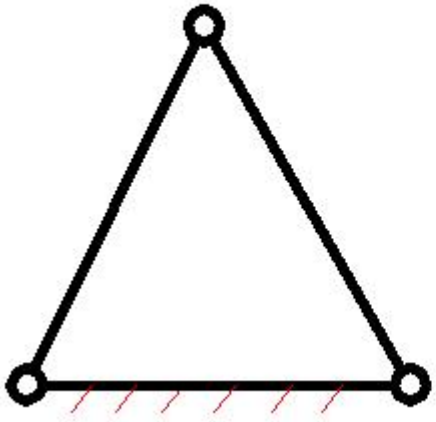


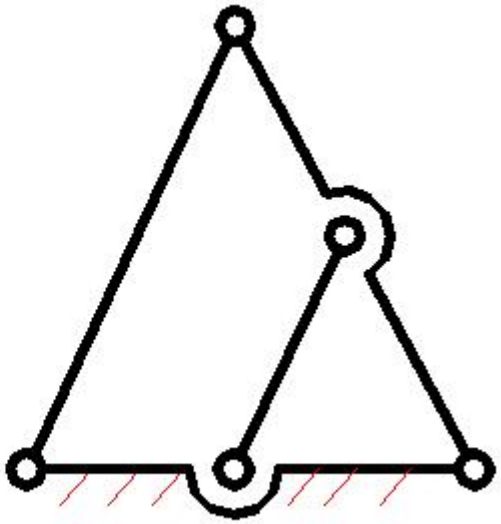
$$F = 3 \times 4 - 2 \times 6 - 0 = 0$$

If F=0, it is a truss(桁架).

构件间没有相对运动
机构→刚性桁架





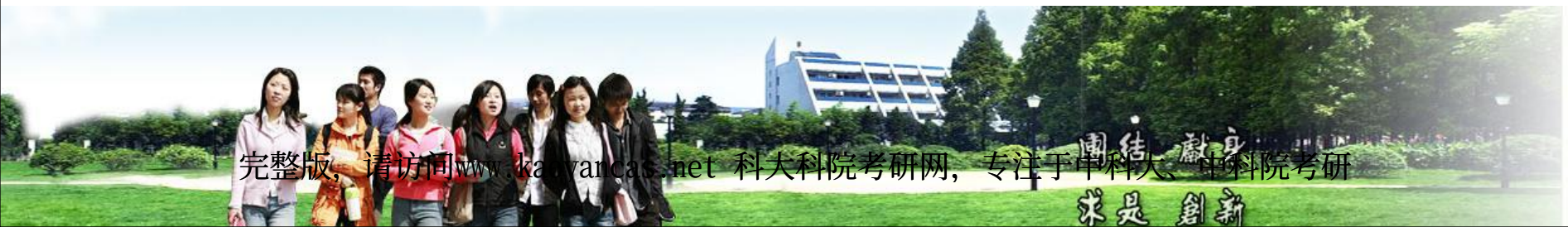


$$F = 3\times 2 - 2\times 3 - 0 = 0$$

刚性桁架

$$F = 3\times 3 - 2\times 5 - 0 = -1$$

（多一个约束）
超静定桁架



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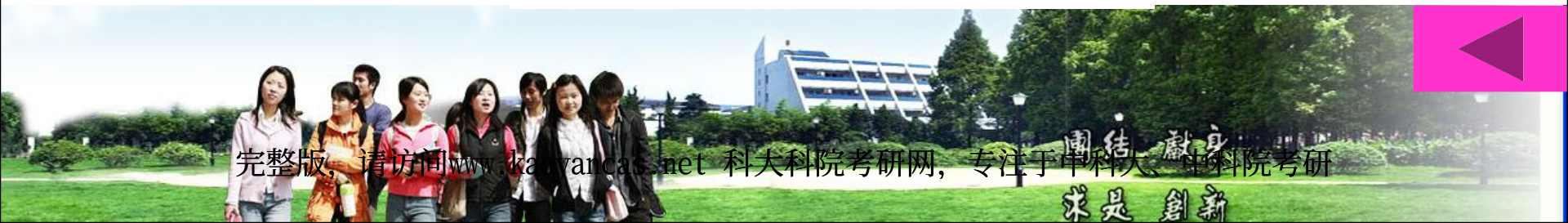
Conclusion:

The DOF of the mechanism is greater than zero. It equals to the number of the driving links→the conditions for a mechanism to have a determined motion.

F≤0，构件间无相对运动，不成为机构。

F>0，
- 原动件数=F，运动确定
- 原动件数<F，运动不确定
- 原动件数>F，机构破坏



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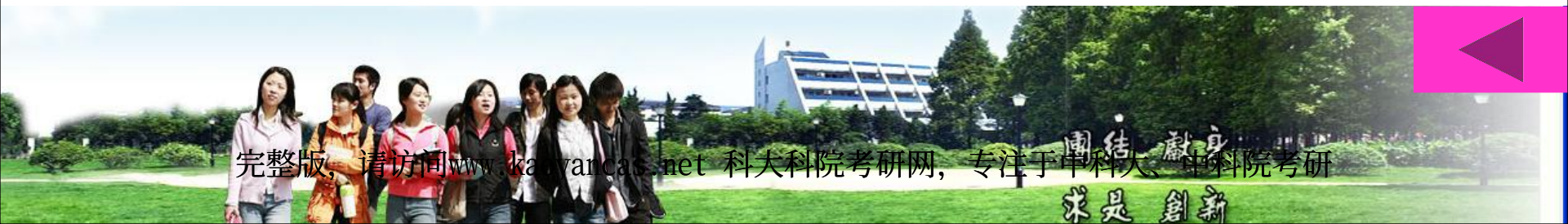
2.5 Points for Attention during the Calculation of DOF

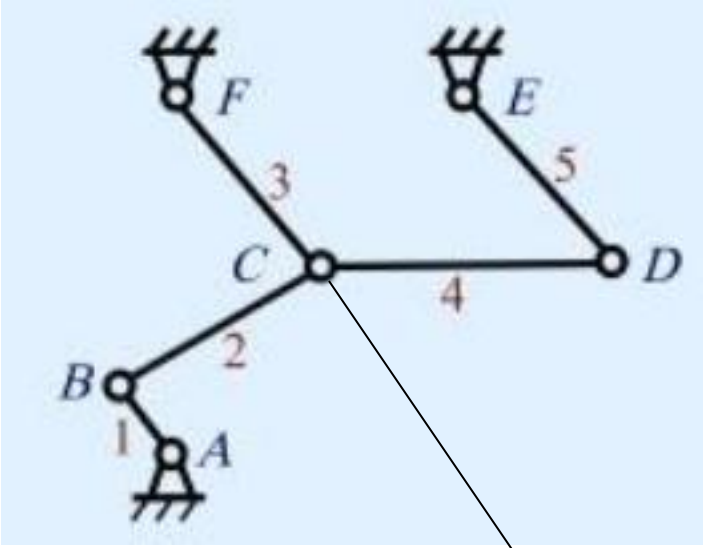
2.5.1 Compound Hinge

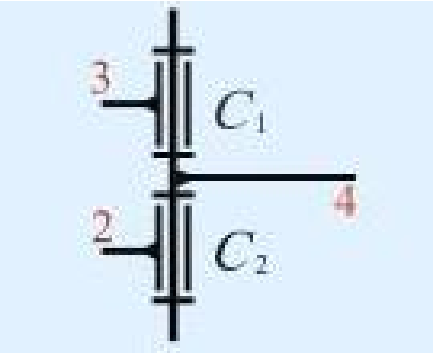
2.5.2 Passive DOF (局部自由度)

2.5.3 Redundant Constraints



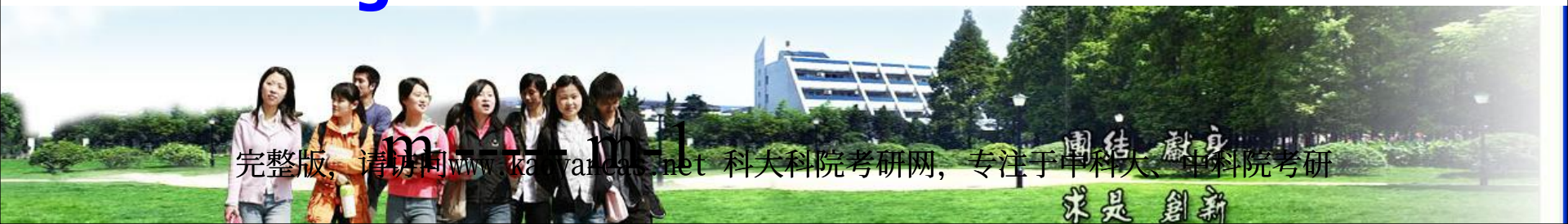
2.5.1 Compound Hinge



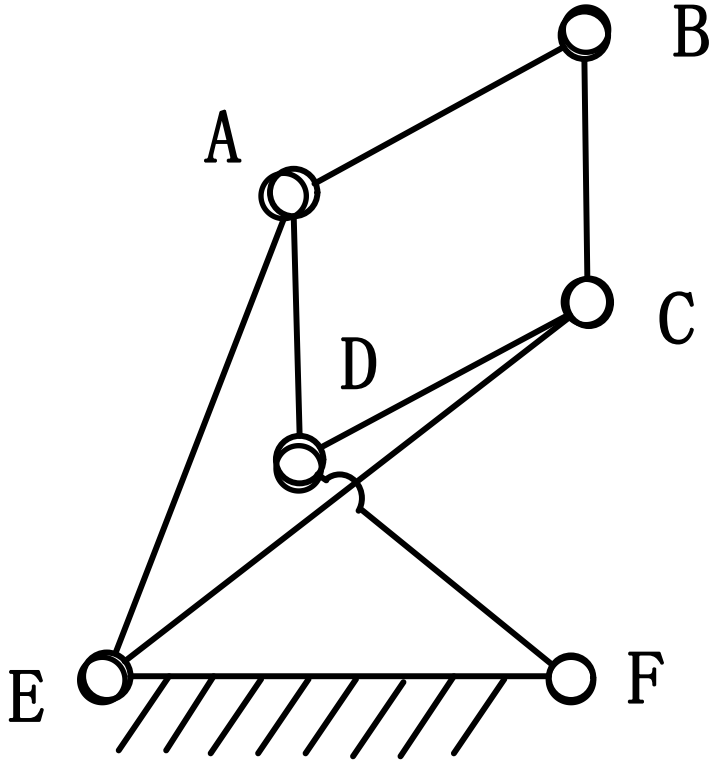


Multiple joints(compound hinge)

The number of revolutes in a compound hinge=one less than the number of links joined at that hinge.

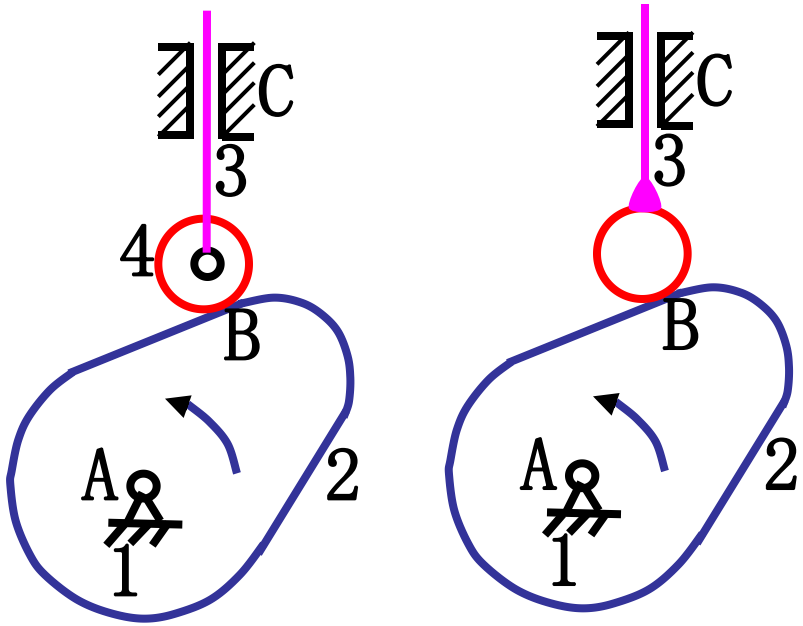


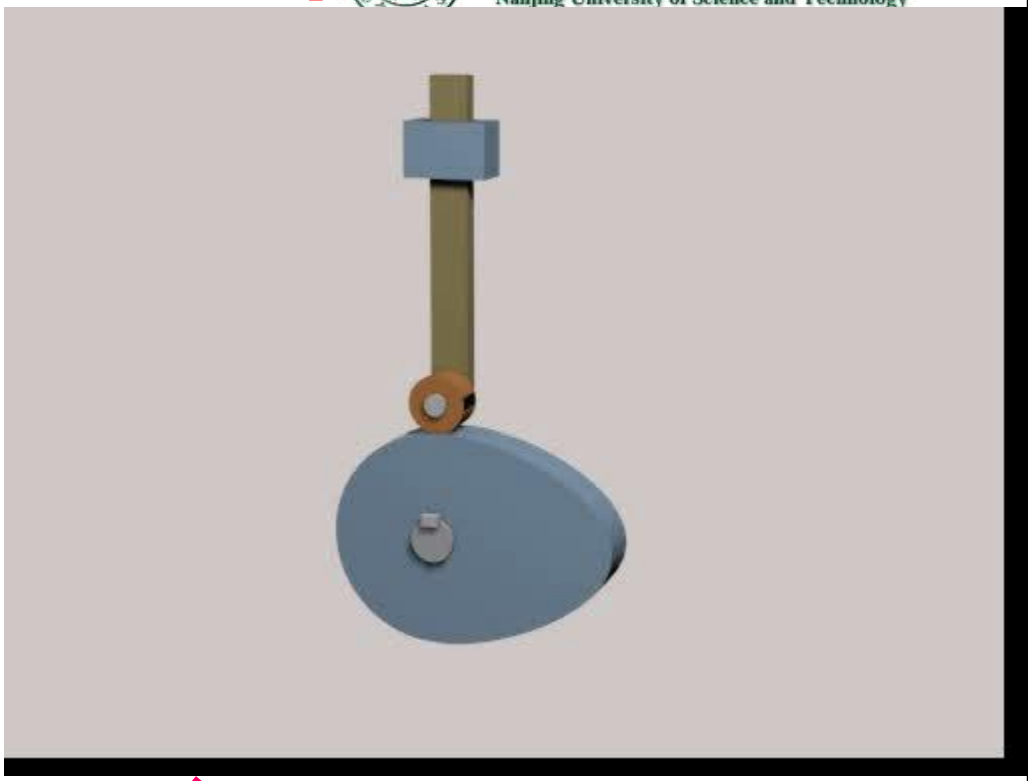
Example:



$$F = 3 \times 7 - 2 \times 10 - 0 = 1$$

2.5.2 Passive DOF (局部自由度)



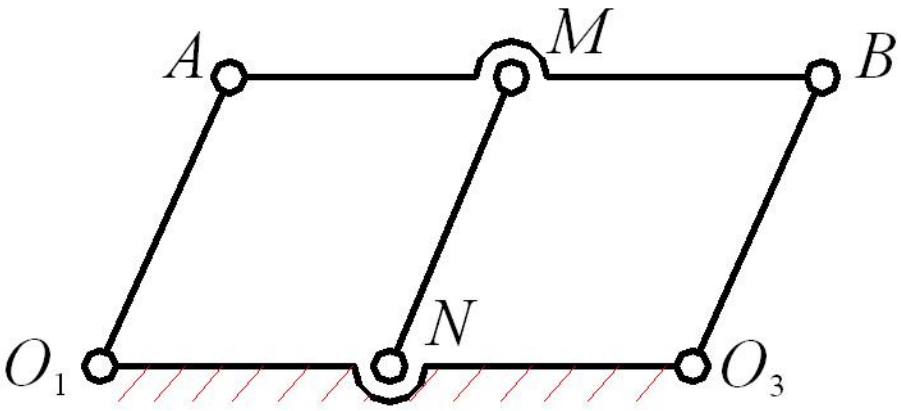


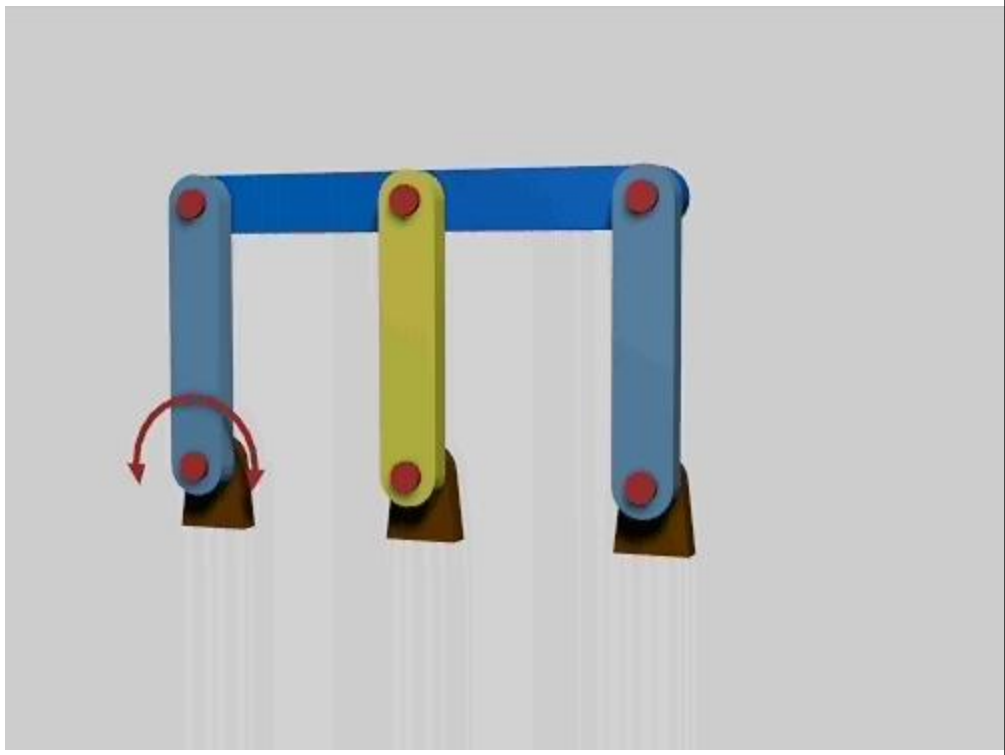
$$F = 3\times3-2\times3-1=2$$

$$F = 3\times2-2\times2-1=1$$

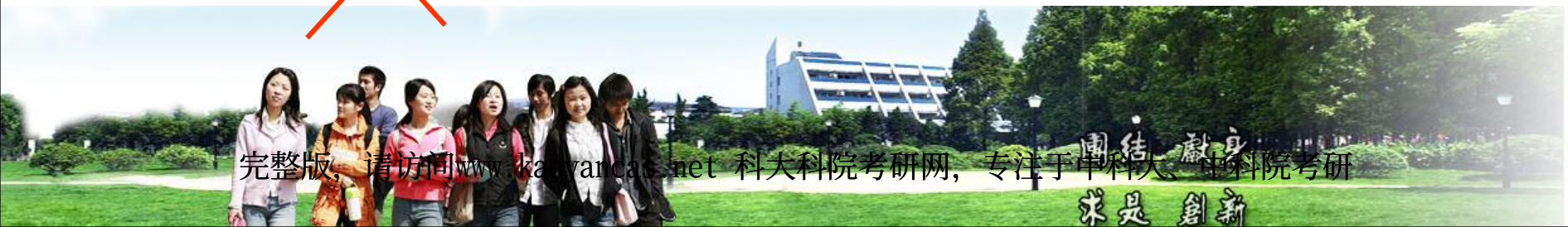
2.5.3 Redundant Constraints

在特殊的几何条件下，有些约束所起的限制作用是重复的，这种不起独立限制作用的约束称为虚约束。





$$F = 3\times 4 - 2\times 6 - 0 = 0$$



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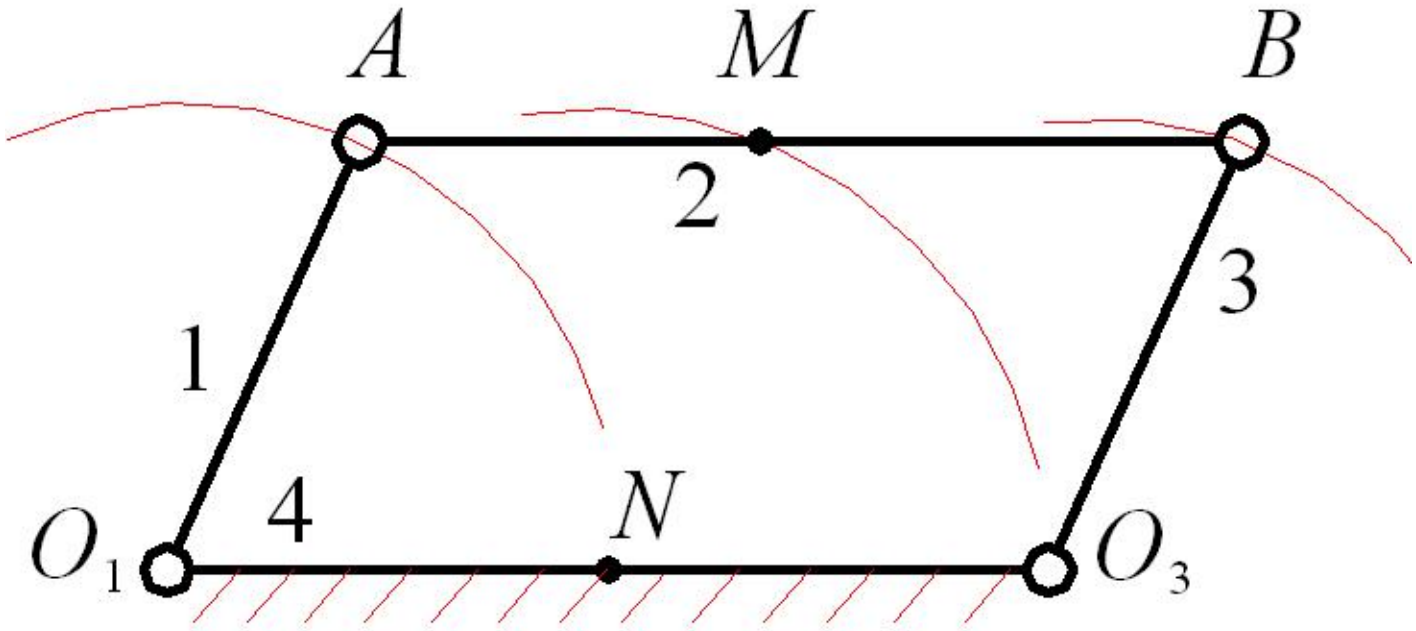
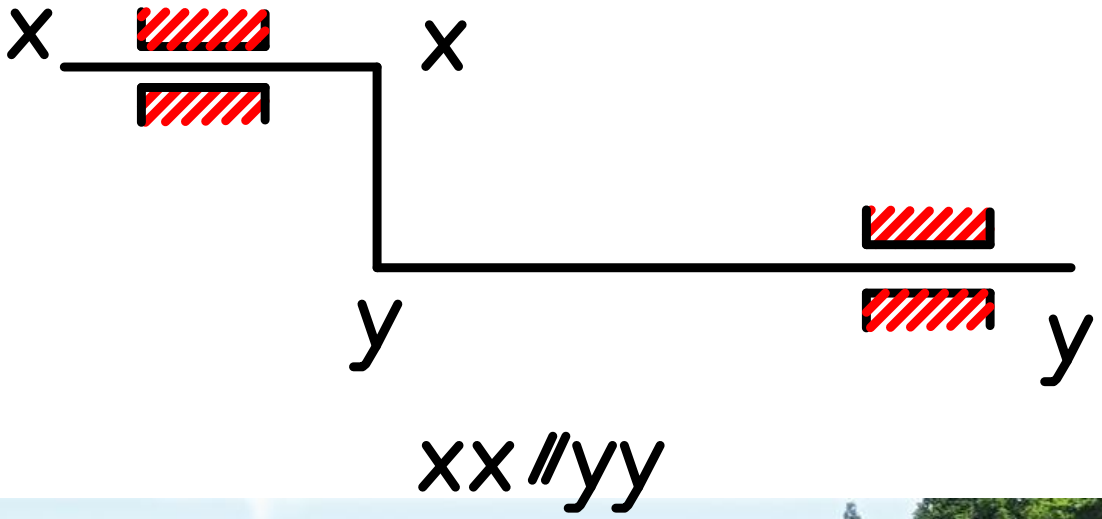
解决方案



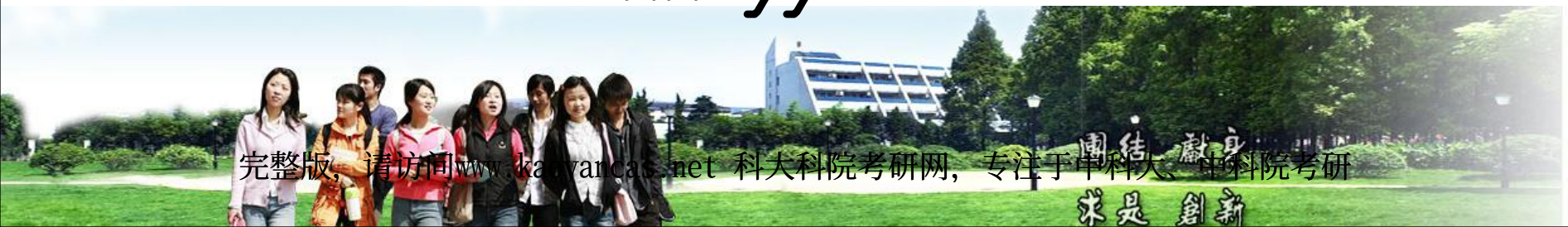
$$F = 3 \times 3 - 2 \times 4 - 0 = 1$$

虚约束常出现的场合

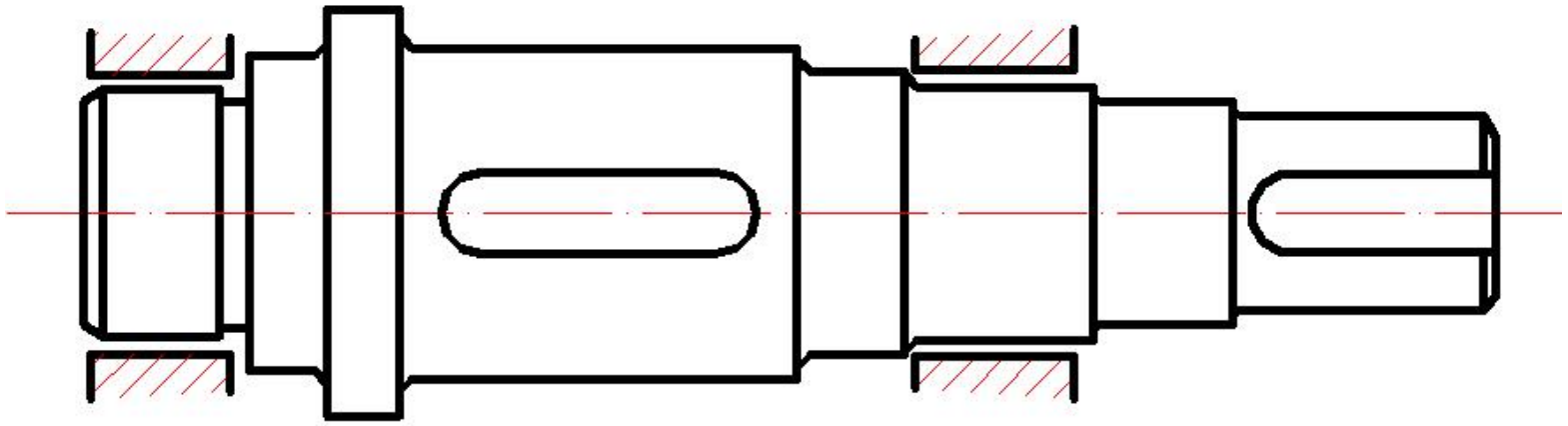
(1) Two links are connected by more than one parallel sliding pairs

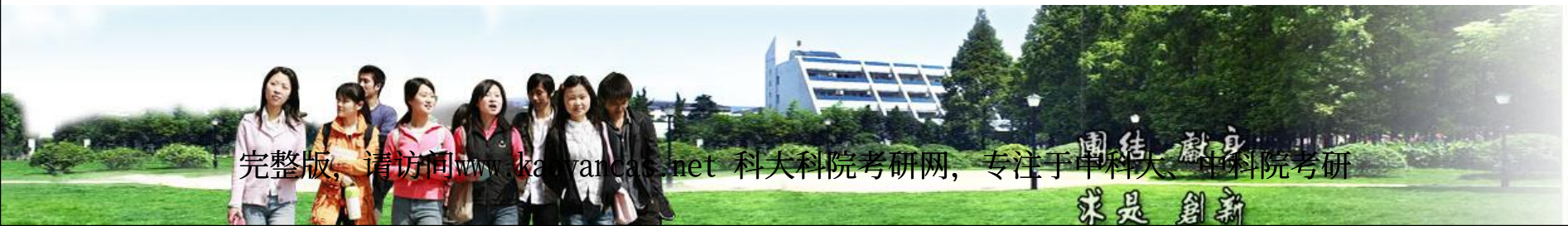


xx∥yy

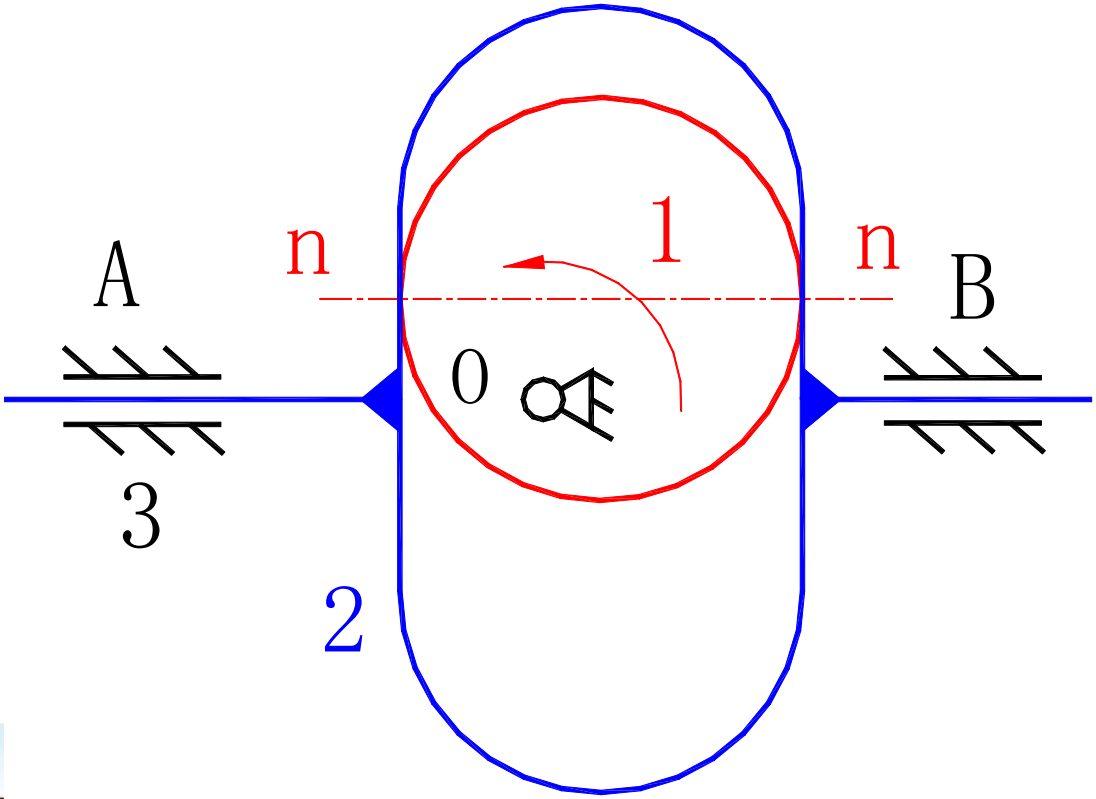


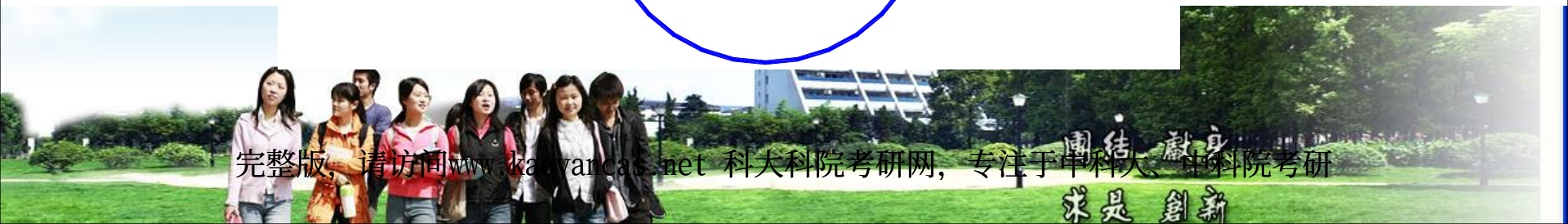
(⑵)Two links are connected by more than one revolute pair whose axes coincide



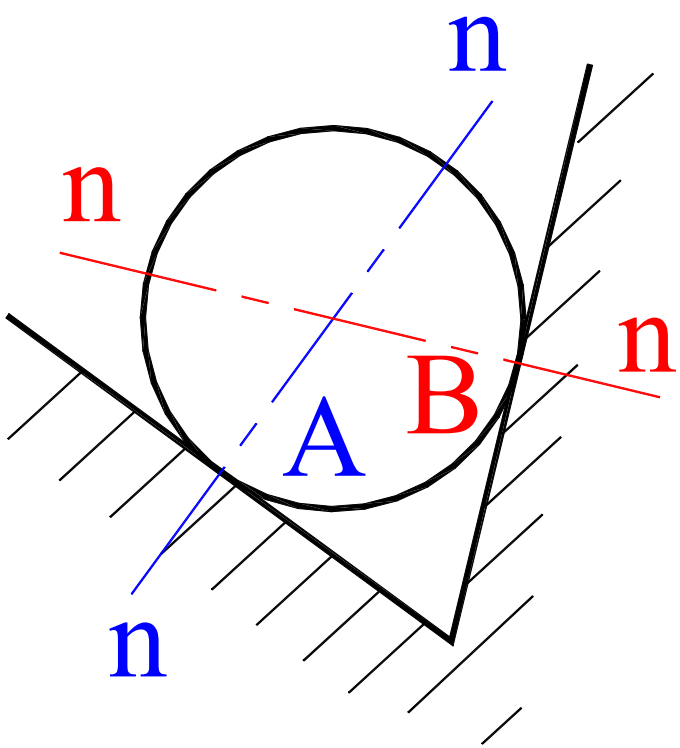


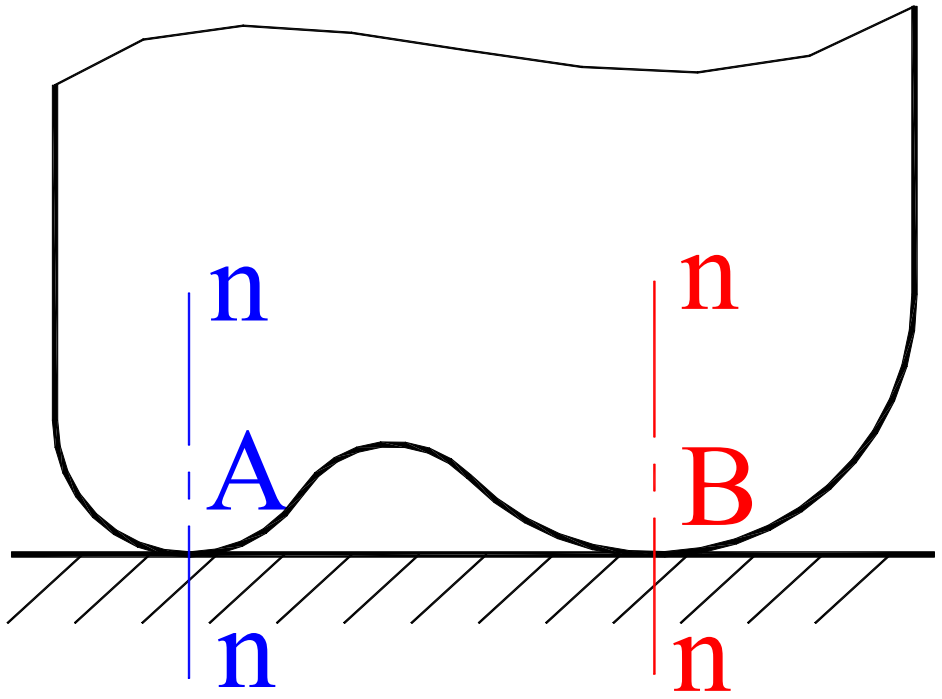
(⑶)Two links are connected by more than one higher pair whose common normals passing through the points of contact coincide.



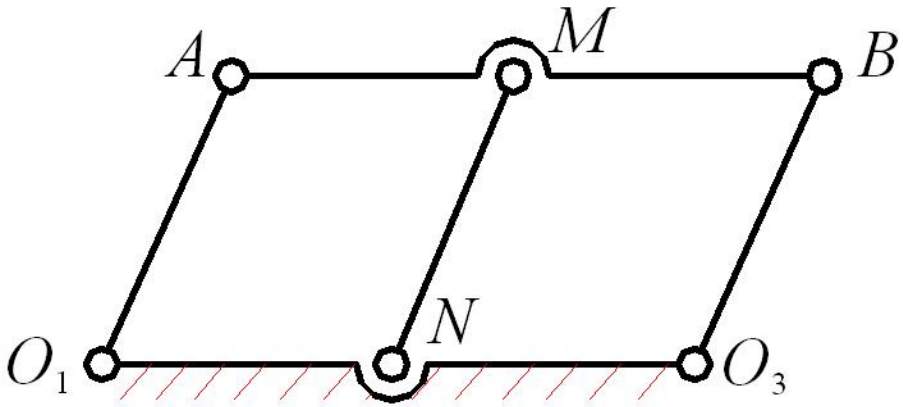


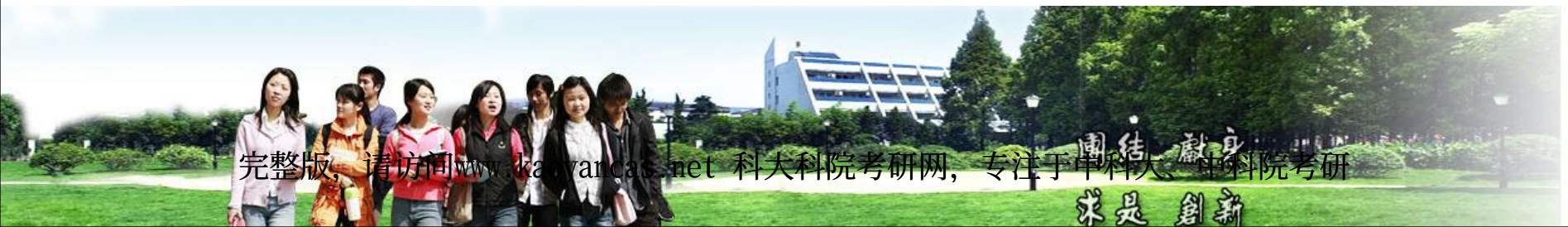
However, if their common normals (公法线) do not coincide(重合), then both higher pairs should be counted.





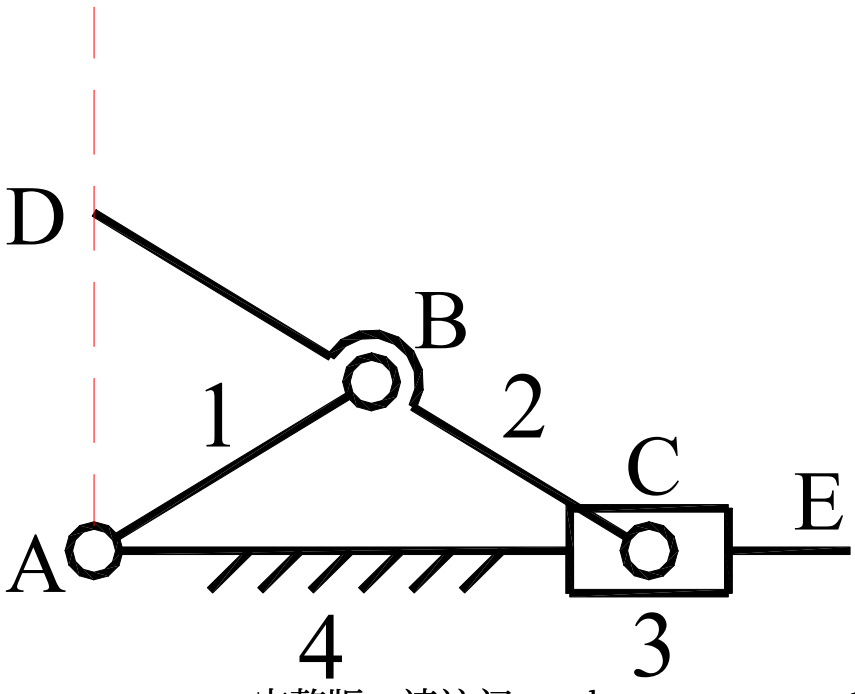
(4) The distance between two points on two links remains constant during the motion of the mechanism,adding one link and two revolutes create a redundant constraints.



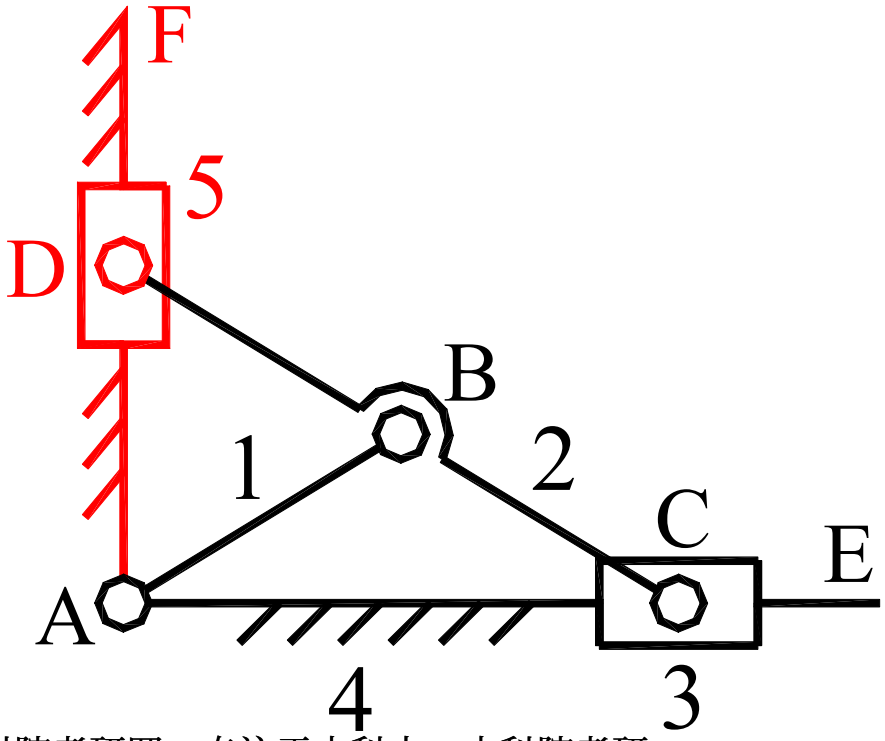


(5) The locus of a point is a straight line,adding one link with one fixed guide way parallel to the line and one revolute with its centre at that point will create a redundant constraint.

$$F=3n-2P_L-P_h=3\times3-2\times4=1$$



AB=BC=BD



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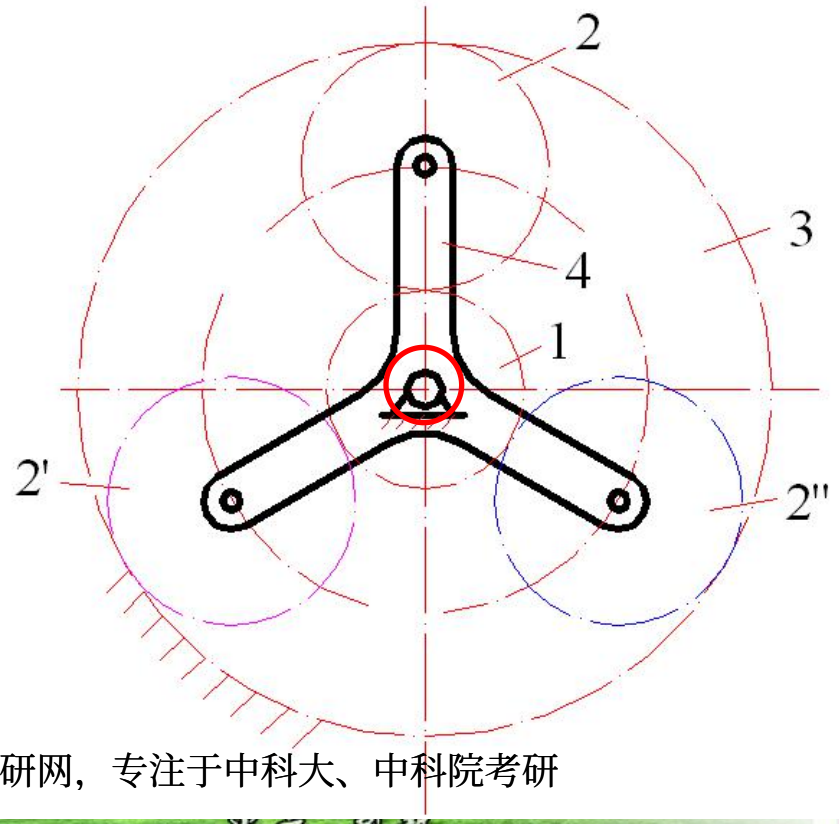
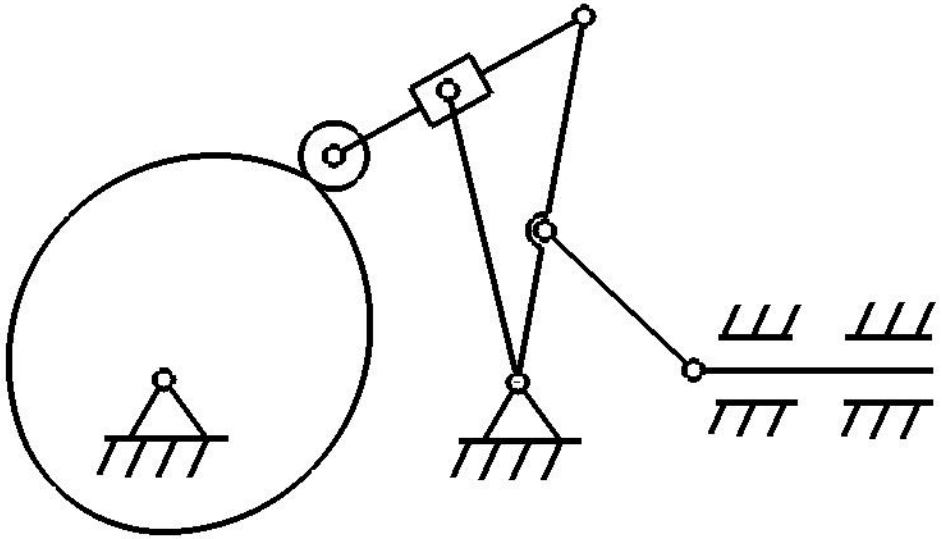
(6) Symmetrical or duplicated structure

In order to transmit(传输) more power, many pinions(小齿轮) are distributed(分布) symmetrically in space. Only one pinion should be counted.

Note：gear 1, gear 3, and link 4 constitute(组成) a compound hinge(复合铰链).

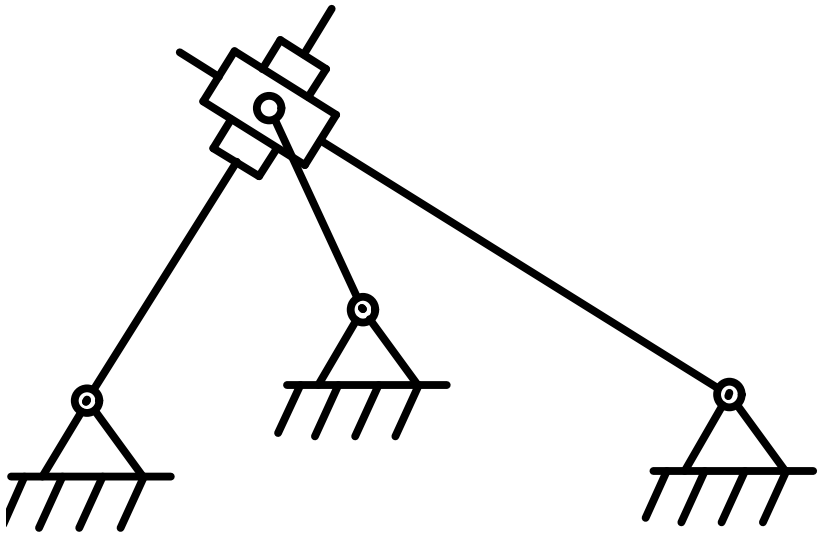


Example2



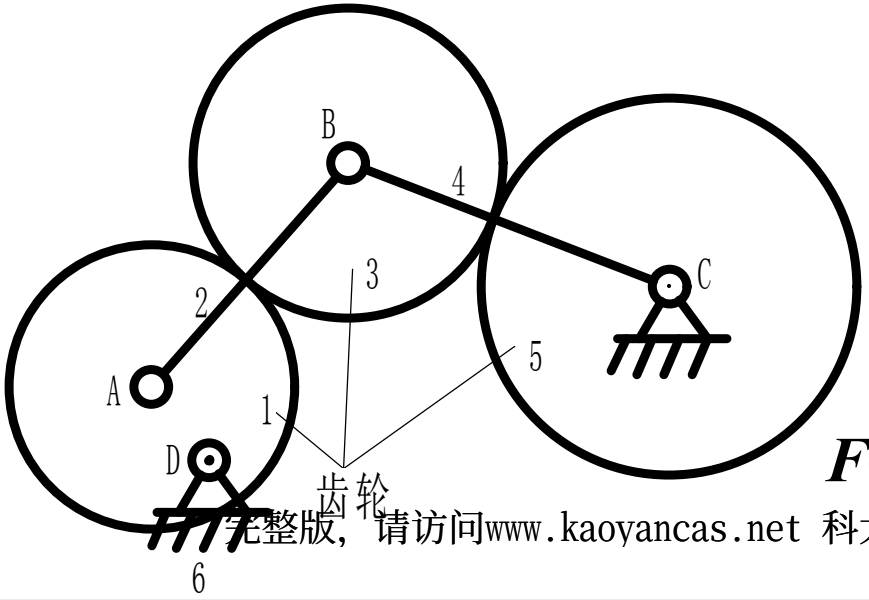
$n=7$，$p_l=9$，$p_h=1$

$F=3n-2p_l-p_h=3\times7-2\times9-1=2$



$n=5$，$p_l=7$，$p_h=0$

$F=3n-2p_l-p_h=3\times5-2\times7-0=1$



$n=5$，$p_l=6$，$p_h=2$

$F=3n-2p_l-p_h=3\times5-2\times6-2\times1=1$

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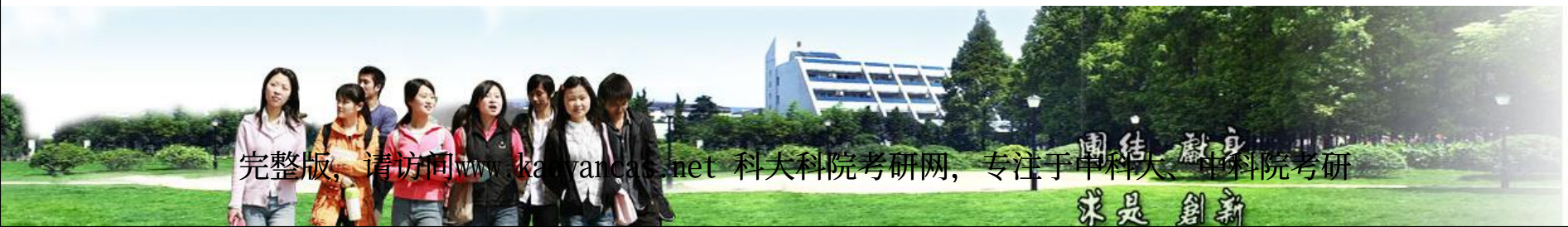
Attention：

(1) Redundant constraints can improve the rigidity(刚度) of a mechanism, improve the force condition in links, etc. and are widely used.

虚约束不影响机构的运动，但可改善构件的受力状况和刚度

(2) "Deleting the redundant constraints during the calculation of DOF" does not mean that the redundant constraints should be omitted(省略) from real mechanisms.

虚约束在计算DOF时去掉，但实际应用不能去掉



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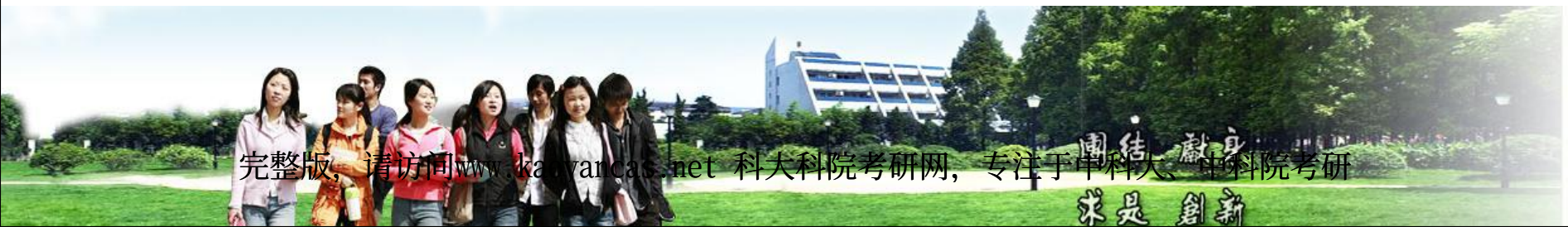
(3) All redundant constraints require some special dimensions. Therefore, attention should be paid to manufacturing(制造) accuracy(精度) when any redundant constraint is used.

虚约束需要满足准确的几何关系，否则便成了实约束



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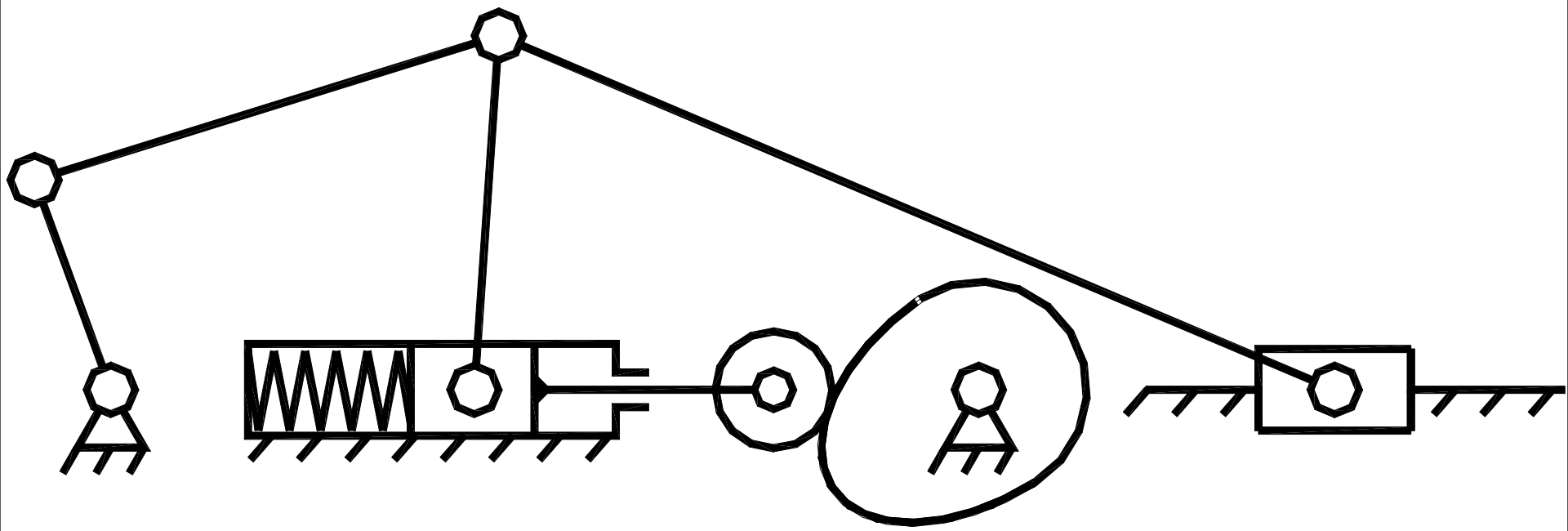
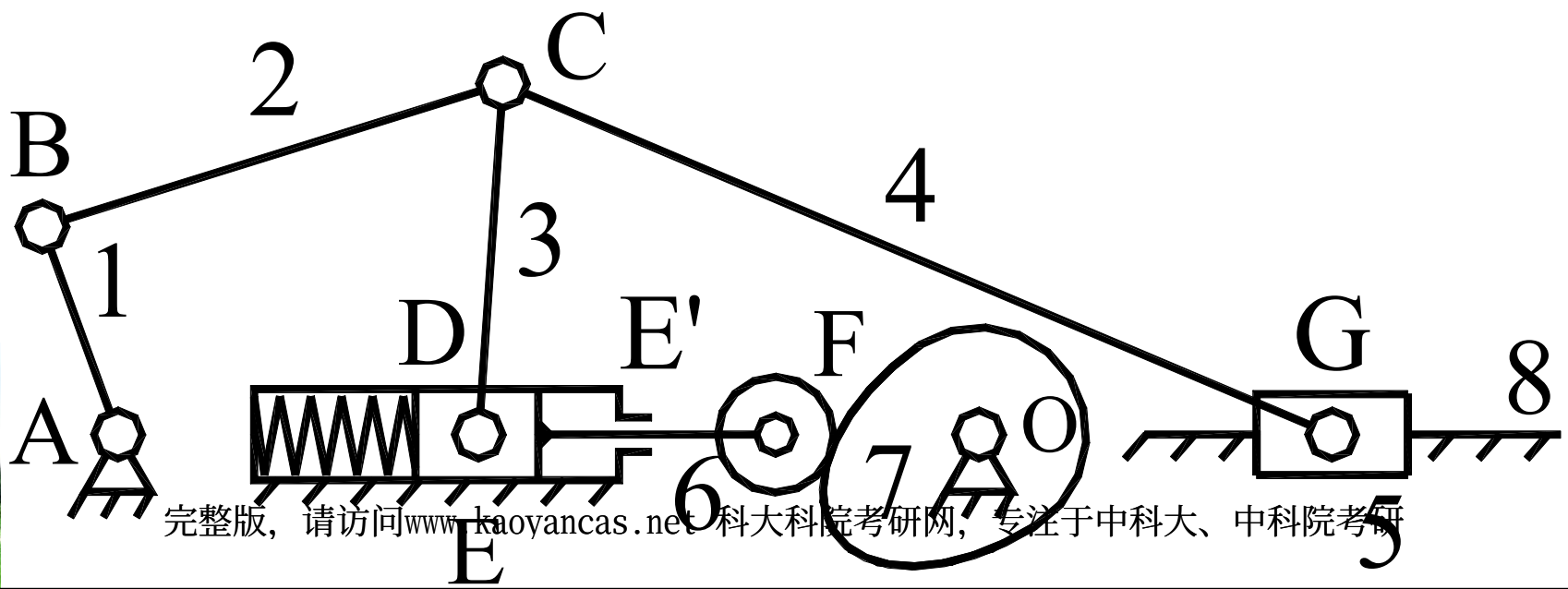
Example1 ： Calculate the DOF of a Mechanism

（先看有无注意事项，复合铰链……，再看有几个构件）



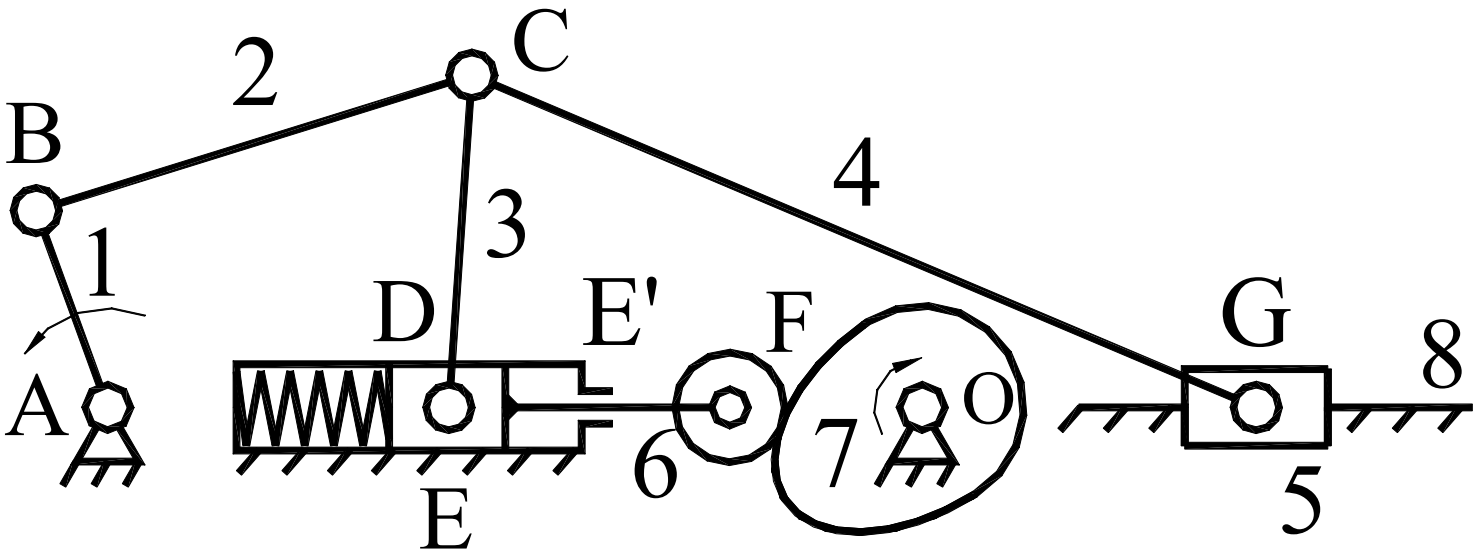
Solution:

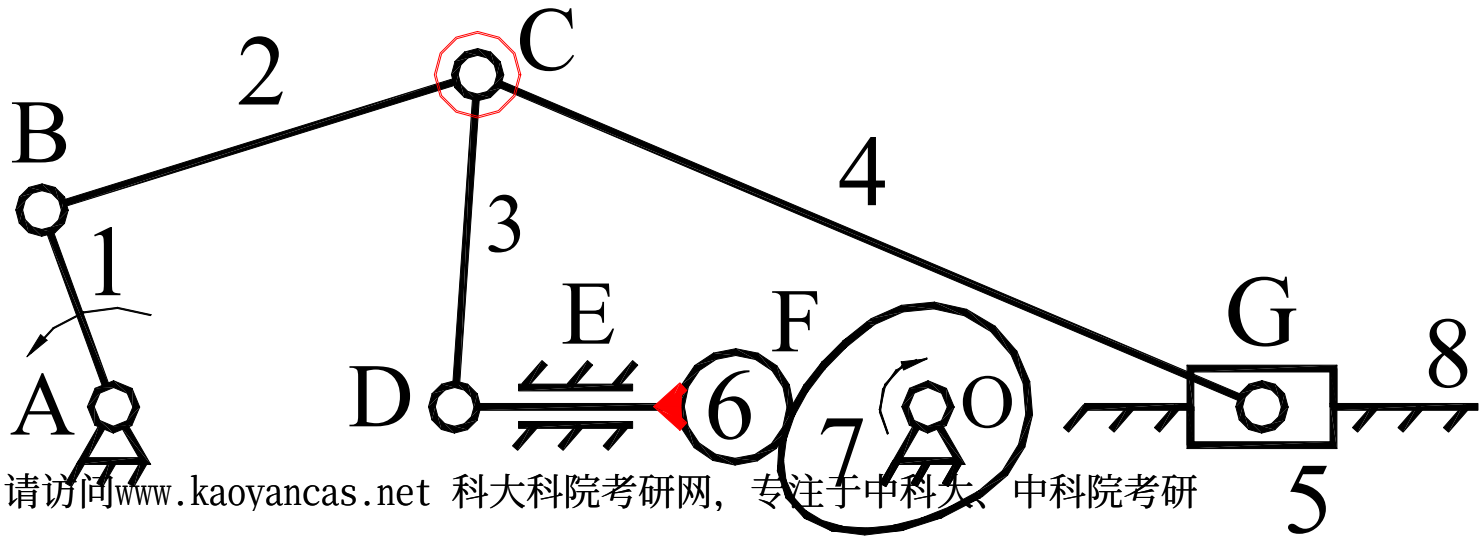
(1)The spring(弹簧) should not be counted.
(2) C is a compound hinge of links 2、3、4。
(3) The roller has a passive DOF(局部自由度).
(4) There are two parallel(平行) sliding pairs E and E′ between the frame 8 and the slider 6. One of the sliding pairs is redundant(多余的).



After the mechanism is redrawn,

F=3n-2PL-Ph=3×7 -2×9 -1=2。Needs two drivers.





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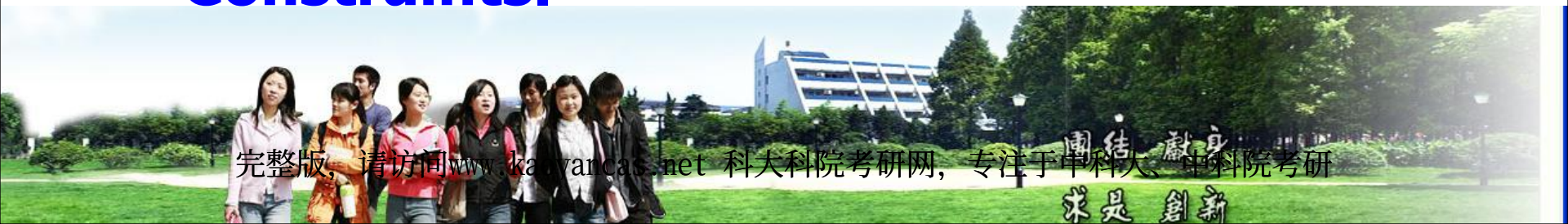
Attentions (this chapter):

- Planar Kinematic Pairs
- Planar Mechanisms
- The Kinematic Diagram of a Mechanism
- Calculate the DOF of the mechanism
- Compound Hinge,Passive DOF, Redundant Constraints.



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Homework

1. Read book p4-23.
2. Exercise p23,2-6:Fig2-43,2-44,2-45,2-46
3. Calculate the DOF p23,2-7:
   Fig2-47,2-50,2-51,2-52